

# Probabilistic Structural Health Monitoring Using Acoustic Emission

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## ABSTRACT

Fatigue crack initiation and growth during the service of aging aircraft are important life-limiting phenomena. In a previous study, a risk prediction and reliability model for naval aircraft has been developed based on fracture mechanics and inspection field data. Despite significant achievements in the study of fatigue cracks using fracture mechanics, it is still of great interest to find practical techniques for monitoring the crack growth using non-destructive inspection and to integrate the inspection results with the fracture mechanics models to improve the predictions. In this paper, a probabilistic damage-tolerance model based on acoustic emission monitoring is proposed to enhance the reliability and risk prediction for structures subject to fatigue cracking. Experiments were carried out to estimate the stress intensity range  $\Delta K$ , during fatigue crack propagation using acoustic emission (AE) inspection. The uncertainty of parameters is captured via probability distribution functions. Bayesian regression technique was used to estimate the marginal and joint probability distributions of model parameters. Finally, an AE-based risk factor is defined as the probability of transitioning from stage II to stage III of fatigue crack growth regime. This transition probability is calculated as the probability that the maximum stress intensity exceeds the

fracture toughness of the material at a given point in time, based on the AE inspection results.

## 1 INTRODUCTION

In recent years, there has been considerable interest in developing risk prediction and reliability models for aging structures such as airframes. (Wang et al., 2009) have proposed a probabilistic model to assess the reliability of aging airframes by predicting the probability that a crack will reach an unacceptable length after specified flight hours. They have also shown (Wang et al., 2008) that using prediction models alone is not sufficient to guarantee the safety of a mission. The objective of this research is to use the information extracted from acoustic emission data to assess the severity of fatigue damage in real-time thereby enhancing the quality of risk predictions.

Over the past 30 years, acoustic emission technology has been developed as a promising and effective non-destructive inspection (NDI) technique capable of detecting, locating and monitoring fatigue cracks in a variety of composite and metal structures such as airframes (Boller, 2001). Acoustic emissions are elastic stress waves generated by a rapid release of energy from localized sources within a material under stress (Mix, 2005). Such emissions often originate from defect related sources such as permanent microscopic deformations within material and fatigue crack extension.

In the present study, acoustic emission technique is used, instead of complex procedures and calculations, to determine the stress intensity range  $\Delta K$  in fatigue crack propagation. The value of  $\Delta K$  depends on the geometry, stress amplitude and the instantaneous crack size. For a given geometry, a large  $\Delta K$  represents either a large crack size and/or a high stress amplitude range applied to the structure. Stress intensity is a parameter

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that can be considered an aggregate driving force for fatigue crack growth. Fracture toughness  $K_{IC}$  on the other hand can be thought of as a measure of material's resistance to stable crack propagation under cyclic loading (Anderson, 1994). The crack growth is stable as long as the stress intensity is less than the fracture toughness of the material. In an attempt to use AE for quantitative health monitoring, we define a risk factor  $R_{AE}$  based on the probability that the maximum stress intensity  $K_{max}$  estimated from AE signals, exceeds  $K_{IC}$ . Due to both epistemic and aleatory uncertainties involved (Modarres et al., 1999) in the estimation process,  $K_{max}$  is best represented by a Probability Density Function (PDF),  $f_{K_{max}}$ . The risk factor is defined accordingly and is presented in Eq. (1):

$$R_{AE} = \Pr(K_{max} > K_{IC}) = 1 - F_{K_{max}}(K_{IC}) \quad (1)$$

Where  $F_{K_{max}}$  is the Cumulative Density Function (CDF) of  $K_{max}$ .

In the next section, the experimental procedures including fatigue testing, crack length measurement and AE monitoring will be explained. In section 3, the correlation between AE signals and  $\Delta K$  is established. Next, Bayesian regression approach is used to find the PDF of  $K_{max}$  and consequently calculate  $R_{AE}$  as a function of AE parameters.

## 2 EXPERIMENTAL PROCEDURE

A PCI-2 AE monitoring system supplied by Physical Acoustic Corporations was used to monitor fatigue crack propagation in a compact tension (CT) specimen (ASTM E647-08, 2008) made of 7075-T6 aluminum alloy. During the test, several AE parameters (e.g. AE hit time, load level, amplitude, absolute energy, etc.) as well as fatigue crack growth data (applied load history, crack size  $a$  and number of elapsed cycles  $N$ ) were recorded. The recorded AE and fatigue data were synchronized on one PC to facilitate further analysis.

### 2.1 Fatigue Testing

Fatigue tests were carried out on standard CT specimen (W=2.5 inch, B=0.125 inch) using a 5 kip MTS machine. The specimen was first fatigue pre-cracked using sinusoidal loading with min-max loading ratio R=0.1 and frequency of 30 Hz until fatigue crack of adequate length and straightness in accordance with ASTM E647 was detected. The main fatigue test was performed at a frequency of 10 Hz using the same R ratio of 0.1. The applied load range was determined according to the material properties and geometry of the test specimen and remained fixed throughout the test. Macro digital photography was used for crack size measurement; high resolution pictures of the specimen

(with a scribed scale attached to it) were automatically taken using time-lapse photography technique. The pictures were post-processed using Image Processing Toolbox in MATLAB to identify the crack tip. The crack length was then measured with an accuracy of 0.01 inch.

### 2.2 Acoustic Emission Monitoring

For AE measurement, a wideband (WB) sensor was clamped on the specimen with silicon grease used as coupling agent. AE signals were first amplified using a 40 dB differential amplifier. A 200 kHz high pass filter was used to filter out the extraneous noise mostly from the MTS machine. Signals with amplitudes exceeding a threshold of 45 dB were transferred to a computer for feature extraction. Table 1 shows the important parameter settings for the AE system.

Table 1: AE Hardware settings

Parameter	Value
Preamplifier	40 dB
Peak Definition Time (PDT)	300 $\mu$ s
Hit Definition Time (HDT)	500 $\mu$ s
Hit Lock Time (HLT)	1000 $\mu$ s
Threshold	45 dB
Sampling rate	5 MSPS
Pre-trigger length	100 $\mu$ s
Hit length	614 $\mu$ s
Analog Filter (low)	200 KHz
Analog Filter (high)	3 MHz

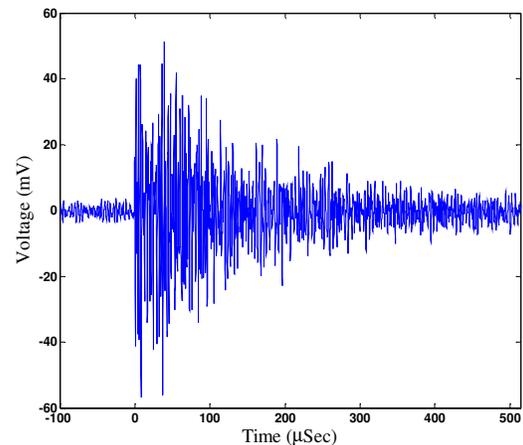


Figure 1: Typical AE signal due to crack growth

Several AE features were calculated by the system and recorded for further analysis. Time domain features included hit time, ringdown count, amplitude, duration,

absolute energy and load level. Frequency domain features included peak frequency and frequency centroid (a measure of average frequency) of the signals. Figure 1 shows a typical AE signal generated during fatigue crack growth.

### 3 RESULTS AND DISCUSSION

#### 3.1 Acoustic Emission Response during Fatigue

Many researchers have studied the correlation between AE parameters and fatigue crack growth behavior (Hamel et al., 1981; Bassim et al., 1994). AE ringdown count  $c$  (number of times the AE signal amplitude exceeds a threshold value) and its derivative, count rate  $dc/dN$ , are two of the most commonly used AE parameters in fatigue. (Bassim et al., 1994) have proposed a relationship which correlates the AE count rate with  $\Delta K$  as follows:

$$\frac{dc}{dN} = B' \Delta K^{\alpha'} \quad (2)$$

Where  $c$  denotes the AE count,  $\Delta K$  is the stress intensity range and  $B'$  and  $\alpha'$  are model parameters which mainly depend on material properties and should be determined experimentally. Our goal is to estimate  $\Delta K$  using AE parameters; therefore we use the inverse of Eq. (2) as follows:

$$\Delta K = B \left( \frac{dc}{dN} \right)^{\alpha} \quad (3)$$

Where  $B = B'^{-1/\alpha'}$  and  $\alpha = 1/\alpha'$ . Linearizing Eq. (3) yields:

$$\ln \Delta K = \alpha \ln \left( \frac{dc}{dN} \right) + \beta \quad (4)$$

Where  $\beta = \ln B$  that will be estimated along with parameter  $\alpha$  using the experimental results.

Signals received during acoustic emission testing are often buried in noise from numerous sources such as surface rubbing at loading pins, noise from the hydraulic loading actuators, internal rubbing of crack surfaces, etc. Researchers (Berkovits and Fang, 1995; Fang and Berkovits, 1993) have proposed different denoising techniques to overcome this shortcoming.

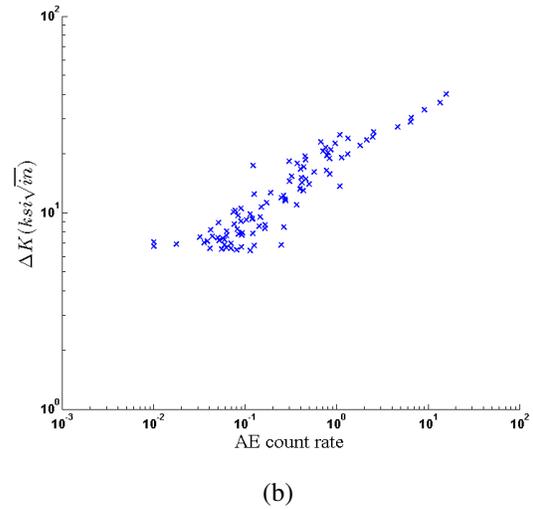
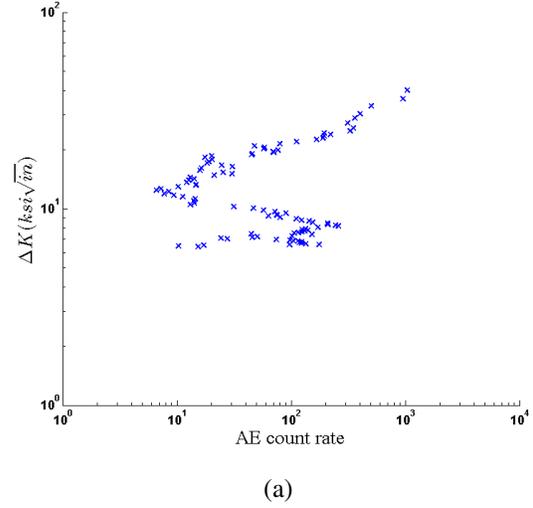


Figure 2: Correlation between AE count rate and  $\Delta K$   
a) before filtration, b) after filtration

Majority of investigators have assumed that only events occurring near the maximum load in a cycle are associated directly with crack extension (Roberts and Talebzadeh, 2003). In the present study, we found that the events (i.e., AE hits) occurring within the top 30% of the peak load have a good correlation with  $\Delta K$  and consequently the crack growth rate. The second criterion used for AE filtration was that the events occurring during the loading portion of a cycle are more likely to be due to crack extension versus those occurring during the unloading part. Figure 2 shows the correlation between  $\Delta K$  and the AE count rate before and after applying these filters.

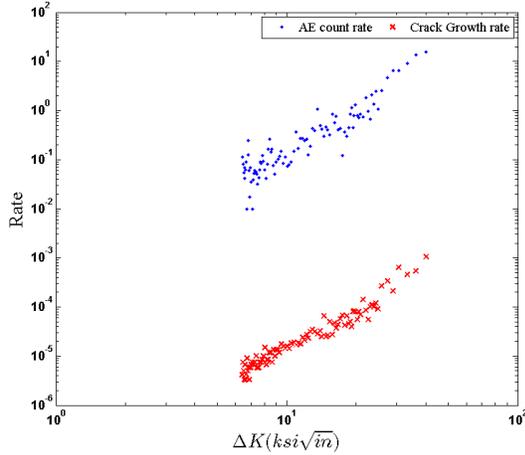


Figure 3: Strong linear correlation between the stress intensity range and both AE count rate and crack growth rate

Figure 3 shows the AE count rate and the crack growth rate on the same plot. Both rates increase linearly with  $\Delta K$  when plotted in log-log scale. This shows how proper feature extraction and filtration of AE signals lead to parameters that can be used to describe the crack growth behavior without a need to measure the crack size. This result is in good agreement with the linear model proposed in Eq. (4).

### 3.2 Probabilistic Reliability Model

In this section, Bayesian regression technique is used to estimate the parameters  $\alpha$  and  $\beta$  of Eq. (4). Rather than relying solely on the best estimate of the parameters and the corresponding confidence intervals, as is the common practice when using Maximum Likelihood Estimation (MLE) and traditional regression techniques, Bayesian estimation provides a reasonable coverage of the uncertainties by calculating the joint probability density function of the model parameters. Another advantage of Bayesian approach is that it preserves the available information in the scatter of the data in the form of posterior probability distributions for the model parameters.

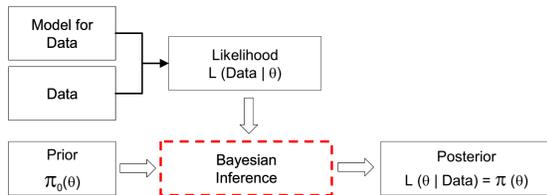


Figure 4: Bayesian Inference Framework (Azarkhail and Modarres, 2007)

In addition, Bayesian inference technique provides a framework for incorporating any additional sources of

knowledge that may be available about the parameters. Possible sources of such information include past experiments, handbook data and expert judgment. See (Azarkhail and Modarres, 2007) for more information on using Bayesian regression technique for uncertainty characterization.

In Bayesian approach to regression, the fitness concept is represented in the probability of occurrence or likelihood form where a larger value of the likelihood function shows a better model fit to the data. One way to define the likelihood function is to use the distribution of error. Here error is defined as the difference between the model prediction and the observed data and can be treated as a random variable. It is assumed that for the best fitted model, the error is normally distributed with mean zero and unknown standard deviation  $\sigma$ . This is equivalent to assuming that the dependent variable is normally distributed with its mean defined by the model prediction and with standard deviation  $\sigma$ . Here we define the likelihood function by assuming that the dependent variable  $\ln \Delta K$  is normally distributed according to Eq. (5).

$$\ln \Delta K \sim N(\mu, \sigma) \quad (5)$$

Where  $\mu = \beta + \alpha \ln(dc/dN)$  is the mean of the distribution which is calculated based on the linear relationship in Eq. (4) and  $\sigma$  is the standard deviation which is an unknown parameter to be estimated along with  $\alpha, \beta$ . The conditional likelihood function can then be formally defined as follows:

$$L\left(\ln \Delta K, \ln\left(\frac{dc}{dN}\right) \mid \alpha, \beta, \sigma\right) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \left( \frac{\ln \Delta K_i - \beta - \alpha \ln\left(\frac{dc}{dN}_i\right)}{\sigma} \right)^2} \quad (6)$$

Bayesian inference starts with an uncertain and subjective belief about the model parameters. This belief is systematically updated using the likelihood function (Eq. (6)) and in light of the available data (i.e. ordered pairs of  $(\ln \Delta K_i, \ln(dc/dN)_i)$ ). In this study, we started with no past experience and therefore no prior information about the distribution of parameters was available. This is reflected in our choice of non-informative (uniform) prior distributions for parameters  $\alpha, \beta$  and  $\sigma$ . If additional information such as similar test results or prior estimates of the model parameters becomes available, an informative prior distribution can be used instead. This will affect the posterior distribution of parameters accordingly. Notice that when uniform priors are used for the parameters, Bayesian and MLE approach will both result in the same best estimate for the parameters but the coverage of the uncertainty over the parameters could be

different. Uncertainty bounds in MLE are estimated using Fisher information matrix with underlying normality assumption for the parameters whereas in Bayesian approach, the uncertainty bounds are derived using the posterior joint distribution of parameters. Figures 5 and 6 show the Bayesian regression results in form of marginal and joint posterior distribution of model parameters.

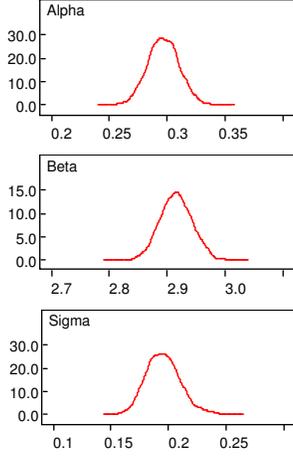


Figure 5: Marginal posterior PDF of model parameters

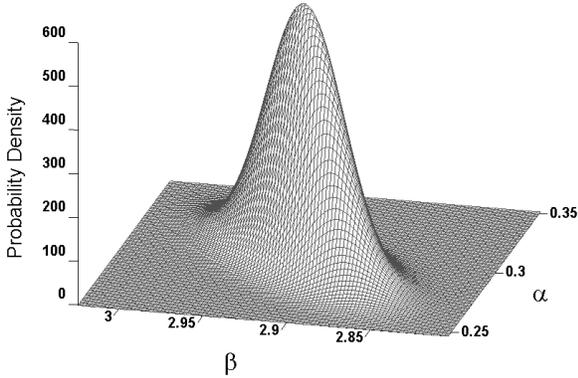


Figure 6: Posterior joint PDF of  $\alpha$  and  $\beta$

In a Bayesian framework, prediction at a given value of the independent variable is based on the *predictive distribution*, that is, the likelihood of the future data averaged over the posterior distribution of parameters as illustrated in Eq. (7).

$$f\left(\Delta K \mid \frac{dc}{dN}\right) = \iiint_{\underline{\theta}} f\left(\Delta K \mid \frac{dc}{dN}, \underline{\theta}\right) \pi(\underline{\theta}) d\underline{\theta} \quad (7)$$

Where  $\pi(\underline{\theta})$  represents the posterior distribution and  $\underline{\theta} = \{\alpha, \beta, \sigma\}$  is the vector of model parameters. It is very difficult and sometimes impossible to solve these equations analytically. Therefore, in practice, numerical approaches such as Monte Carlo based

methods are used to calculate these multidimensional integrals. In this approach, the characteristics of distributions are estimated by generating sufficient number of statistical samples from them. Here we use samples from the posterior joint distribution of model parameters along with Eq. (4) to estimate the distribution of  $\Delta K$  for a given value of  $dc/dN$ .

As the final step to develop an AE-based health monitoring framework, Eq. 1 is used to find the instantaneous risk factor based on the conditional distribution of  $K_{max}$  and the value of fracture toughness  $K_{IC}$ . The risk factor is defined as the probability that the maximum stress intensity exceeds the fracture toughness of the material which results in unstable crack growth and ultimately failure. For a given AE count rate, the corresponding PDF of  $\Delta K$  is found from Eq. (7). The distribution of  $K_{max} = \Delta K/(1-R)$  can then be easily obtained for a known loading ratio  $R$ .

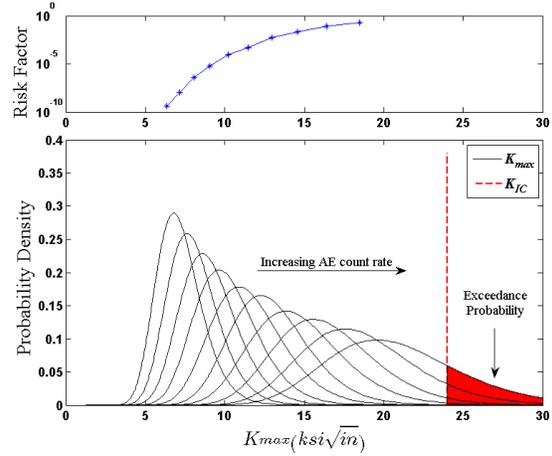


Figure 7: PDF of  $\Delta K$  as the AE count rate increases (bottom), Increasing trend in risk factor (top)

Figure 7 shows the conditional distribution of  $K_{max}$ , estimated from the AE data in Figure 2b. Notice how this distribution shifts to the right as the AE count rate increases. This figure also illustrates the increasing trend in  $R_{AE}$  as the AE count rate and  $K_{max}$  increase throughout the experiment. By monitoring the acoustic emissions from a structure, the proposed approach enables us to estimate, at a given point in time, the probability that the crack growth transitions to the unstable regime and ultimately leads to failure.

In this study a deterministic  $K_{IC}$  value is assumed for simplicity but if additional data about the statistical distribution of  $K_{IC}$  becomes available, the methodology presented here can readily calculate the risk factor accordingly.

#### 4 CONCLUSION

A damage-tolerance reliability model for structural health monitoring was presented in this paper. Experiments were carried out to use AE inspection to estimate the stress intensity range  $\Delta K$  during fatigue crack propagation in a standard CT specimen. Acoustic emission signals were properly filtered and features relevant to fatigue crack growth were extracted. The linear model proposed in the literature for  $\ln \Delta K$  versus  $\ln(dc/dN)$  was confirmed using experimental data. Bayesian regression was used to estimate the marginal and joint probability distributions of model parameters. Next, conditional PDF of  $\Delta K$  given the AE count rate was calculated. Finally, a risk factor  $R_{AE}$  is defined based on the probability that  $K_{max}$  exceeds the fracture toughness of the material  $K_{IC}$  given the AE inspection results. There is room for several improvements in this study: The approach proposed here is also applicable to the case of random amplitude loading when revised to account for the variability in the applied loading. Also, AE filtration and feature extraction can be done in a more sophisticated manner by wavelet analysis and by taking into account more time and frequency domain AE parameters.

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#### NOMENCLATURE

$\Delta K$	stress intensity range
$K_{max}$	maximum stress intensity
$K_{IC}$	fracture toughness
$dc/dN$	acoustic emission count rate
$R_{AE}$	risk factor based on acoustic emission
$\alpha, \beta, \sigma$	model parameters

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