A Prognostic Health Management Based Framework for Fault-Tolerant Control

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ABSTRACT

This paper presents one approach in developing a PHM-based reconfigurable controls framework. A low-level reconfigurable controller is defined as a time-varying multi-objective criterion function and appropriate constraints to determine optimal set-point reconfiguration. A set of necessary conditions are established to ensure the stability and boundedness of the composite system. In addition, the error bounds corresponding to long-term state-space prediction are examined. From these error bounds, the point estimate and corresponding uncertainty boundaries for the remaining useful life (RUL) estimate are obtained. Finally, results are obtained for an avionics grade triplex-redundant electro-mechanical actuator (EMA) with a specific fault mode; insulation breakdown between winding turns in a brushless DC (BLDC) motor is used as a test case for the fault-mode.

1 INTRODUCTION

The emergence of complex and autonomous systems, such as modern aircraft, unmanned aerial vehicles (UAVs) and automated industrial processes is driving the development and implementation of new control technologies aimed at accommodating incipient failures to maintain system operation during an emergency. A prognostics health management (PHM) based fault-tolerant control architecture can increase safety and reliability by detecting and accommodating impending failures thereby minimizing the occurrence of unexpected, costly and possibly life-threatening mission failures; reduce unnecessary maintenance actions; and extend system availability / reliability.

The primary motivation for this research topic emerged over the need for improved reliability and performance for safety critical systems, particularly in aerospace related applications. Fatal accidents in the worldwide commercial jet fleet during the years 1987-2005 were due primarily to (i) controlled flight into terrain, (ii) loss-of-control in flight and (iii) system/component failure or malfunction (Darby, 2006). In a coordinated effort to improve aviation safety, industry and government worked together to reduce the number of fatal commercial aircraft accidents, which dropped by 65% during the period of 1996-2007 (Wald, 2007). As a result of this effort, accidents due to controlled flight into terrain have been virtually eliminated through the addition of various safeguards, but the same cannot be said for accidents due to loss-of-control in flight and system/component failure or malfunctions. System/component failure and malfunctions are recognized as contributing factors to aircraft loss-of-control in flight, so safeguarding against such events will reduce the number of fatal accidents in the two top accident categories (ii) and (iii) respectively.

The remainder of this document is organized as follows. Section 2 presents a literature review for fault detection and diagnosis, long-term prognosis predictions and fault tolerant control strategies. Section 3 defines the FTC architecture. Section 4 studies the stability and boundedness of the reconfigured system and the RUL prediction. Section 5 provides general design guidelines and demonstrates the reconfigurable control algorithms on an EMA. Finally, Section 6 summarizes the findings and future work.

2 LITERATURE REVIEW

According to the NASA ASP IVHM program, the following enabling technologies are necessary before prognosis based control can be considered: fault detection, fault diagnosis and failure prognosis (Srivastava, Mah, & Meyer, 2008). The section concludes with a brief overview of FTC strategies.
2.1 Fault Detection and Diagnosis (FDD)

Over the past three decades, the growing demand for reliability, maintainability, and survivability in dynamic systems has drawn significant research in FDD. Historically, FDD has been used in FTC to retrieve fault information from the system for use in a control recovery strategy and procedure, which is commonly referred to as reconfiguration. Preliminary research by Jiang & Patterson (Jiang & Zhao, 1997; Patterson, 1997) demonstrated that state estimation based schemes are most suitable for fault detection since they are inherently fast and cause a very short time delay in real-time decision making. However, the information from state estimation based algorithms may not be detailed enough for subsequent control system reconfiguration. Work presented by Wu and Zhang (N. E. Wu, Zhang, & Zhou, 2000; Y. M. Zhang & Jiang, 2002) recommends that parameter estimation schemes be used for control reconfiguration and state estimation based schemes for FDD. A unified approach to state estimation/prediction and parameter estimation/identification for FDD using particle filtering was thoroughly studied by M. Orchard (Orchard, 2007).

2.2 Failure Prognosis & Long-Term Prediction

The term prognosis has been used widely in medical practice to imply the foretelling of the probable course of a disease. In the industrial and manufacturing fields, prognosis is interpreted to answer the question, “What is the RUL of a machine or component once an impending failure condition is detected, isolated, and identified?” Within the context of this work, prognosis is defined as (Vachtsevanos, Lewis, Roemer, Hess, & Wu, 2006).

Definition 1 (Prognosis). The ability to predict accurately the RUL of a failing component or subsystem.

Definitions for failure, probability of failure and RUL must be well established before continuing the discussion on prognosis. First, the notion of a failure is defined.

Definition 2 (Failure). An event that corresponds to the fault-dimension, L, entering an unwanted range, or hazard-zone. The hazard-zone is defined by the upper and lower bounds, \( H_{ub} \) and \( H_{lb} \), respectively.

The boundaries of the hazard zone are design parameters related to the false-alarm rate (type I error). It should be recognized any discussion regarding a failure over a future time horizon \( t > t_0 \) is stochastic in nature. Instead, the probability of failure should be used.

Definition 3 (Probability of Failure). The probability of a failure occurring at some time \( t \), represented as,

\[
P_{\text{failure}} (t) = p (H_{lb} \leq L (t) \leq H_{ub}),
\]

(1)

where \( p \) is a probability density function (pdf).

Figure 1 illustrates the predicted fault growth of a system where a fault is detected at time \( t_{\text{detect}} \) and a prediction of the RUL is made at time \( t_{\text{prognosis}} \). The boundaries of the hazard-zone are defined by \( H_{lb} \) and \( H_{ub} \). The probability that a failure occurs outside this boundary is defined as the false-alarm rate, \( \alpha \). The time corresponding to each predicted fault trajectory in the hazard-zone is represented as a distribution on the time-axis. The upper and lower RUL boundary values that encompass a CL of \( 1 - 2\beta \) are represented as \( t_{\text{RUL}(ub)} \) and \( t_{\text{RUL}(lb)} \), accordingly. The width of the corresponding confidence interval is defined as,

\[
\epsilon_{\text{RUL}} \triangleq t_{\text{RUL}(ub)} - t_{\text{RUL}(lb)}.
\]

(6)

Finally, its often convenient to describe the minimum time-horizon (or RUL) corresponding to a failure with a particular level of certainty, represented by the symbol \( t_{\text{RUL}(ib)} \).

Definition 4 (Remaining Useful Life (RUL)). The amount of time before a failure occurs at the initial time of prediction, \( t_0 \). The time corresponding to the probability of failure can be expressed as,

\[
t_{\text{RUL}(ib)} (t_0) \triangleq \min (t^*) \quad \text{s.t.} \quad p_{\text{failure}} (t^* | t_0) \geq \beta,
\]

(2)

where \( t^* \in (t_0, \infty) \) and \( 0 < \beta < 1 \). The symbols \( t_0 \) and \( \beta \) refer to the initial prediction time and the type-II error associated with the prediction accordingly.

Sometimes the term confidence level is used instead of the type-II error, which is defined next.

Definition 5 (Confidence Level (CL)). Let the upper RUL boundary, \( t_{\text{RUL}(ub)} \), predicted at time \( t_0 \) be defined as,

\[
t_{\text{RUL}(ub)} (t_0) \triangleq \min (t^*) \quad \text{s.t.} \quad p_{\text{failure}} (t^* | t_0) \leq 1 - \beta.
\]

(3)

where \( t^* \in (t_0, \infty) \). Then the CL is defined by the following probability,

\[
\text{CL} = \int_{t_{\text{RUL}(ib)}}^{t_{\text{RUL}(ub)}} p_{\text{failure}} (t^* | t_0) \, dt^*.
\]

(4)

Additionally, CL is related to \( \beta \) by,

\[
\text{CL} = 1 - 2\beta.
\]

(5)
Several approaches to prognosis have been investigated in recent years, such as model-based (Yu & Harris, 2001; Paris & Erdogan, 1963), data-driven (Schwabacher, 2005), hybrid methods and particle filtering (Orchard, 2007; Orchard, Kacprzynski, Goebel, Saha, & Vachsevanos, 2009).

2.3 Fault–Tolerant Control (FTC) Strategies

Modern systems rely on sophisticated controllers to meet increased performance and safety requirements. A conventional feedback control design for a complex system may result in unsatisfactory performance, or even instability, in the event of malfunctions in actuators, sensors or other system components. To overcome such weaknesses, new approaches to control system design have been developed in order to tolerate component malfunctions while maintaining desirable stability and performance properties. According to Y. Zhang (Y. Zhang & Jiang, 2003), FTC is defined as,

Definition 6 (Fault-Tolerant Control (FTC) Systems). Control systems that possess the ability to accommodate system component failures automatically [while] maintaining overall system stability and acceptable performance.

Traditionally, FTC systems are classified into two categories: passive and active (Y. Zhang & Jiang, 2008).

2.3.1 Passive Fault-Tolerant Control Systems (PFTCS)

Historically, when fault tolerance was an issue, controllers were designed targeting selected faults with specific control actions to mitigate the risk of impending failures (Isermann, 1984). Within such passive approaches, no fault information is required and robust control techniques are employed to ensure the closed-loop system remains insensitive to specific anticipated faults (Zhenyu & Hicks, 2006). The most common and widely studied PFTCS is robust control. Although PFTCS are widely used, they lack an active reconfiguration of the control law thus disallowing use of any external information such as FDD and prognostics.

2.3.2 Active Fault-Tolerant Control Systems (AFTCS)

AFTCS react to system component failures by reconfiguring control actions to maintain stability and acceptable system performance. AFTCS FTC methodologies typically have two main objectives: FDD and control reconfiguration (Rausch, Goebel, Eklund, & Brunell, 2007). Several authors have reported on the problem of FDD (Filippetti, Franceschini, Tassoni, & Vas, 2000; Kleer & Williams, 1987). In such control systems, the controller compensates for the effects of faults either by selecting a pre-computed control law or by synthesizing a new control scheme on-line (Skormin, Apone, & Dunphy, 1994; Willsky, 1976). An AFTCS consist of a reconfigurable controller, a FDD scheme and a reconfiguration mechanism (B. Wu, Abhinav, Khawaja, & Panagiotis, 2004). Types of AFTCS include adaptive robust control (Saberi, Stoorvogel, Sannuti, & Niemann, 2000), expert control (Isermann, 1997; Levis, 1987; N. E. Wu, 1997), optimal control (Bogdanov, Chiu, Gokdere, & Vian, 2006; Garcia, Pret, & Morari, 1989; Kwon, Bruckstein, & Kailath, 1983), and hybrid control. Particular interest is the work by (Bogdanov et al., 2006) and (Monaco, D.G., & Bateman, 2004) which introduces prognostic information into a control law using model predictive control. For systems where on-line computation is feasible, MPC has proved quite successful (Richea, 1993; Richea, Rault, Testud, & Papon, 1978). Monaco et al. (Monaco et al., 2004) demonstrated an MPC based framework used to retrofit the F/A-18 fleet support flight control computer (FSFCC) with an adaptive flight controller.

3 Control Architecture

The problem of incorporating prognosis in a control system can be approached in a variety of ways. The efficacy of any one approach depends on the problem formulation and the specific application. Therefore, fixed performance criteria are necessary to compare any two designs. In the scope of this work, the controller performance criteria are determined by the ability to prevent a failure while minimizing the impact on overall system performance over a well-defined time horizon.
3.1 Qualitative Example

Consider the plots in Figure 2 for the fault dimension, \( L \), probability of failure, \( p_{\text{failure}} \) and tracking error, \( e \), versus time for three different scenarios. The illustration of this example is simplified by considering a single-input single-output (SISO) case. Let the symbols \( e_{\text{min}} \) and \( e_{\text{max}} \) represent the lower confidence bound of the RUL. In scenario (a) the performance criteria is not relaxed and the RUL is not achieved. That is, the probability of failure exceeds \( \beta \) before time \( t_{\text{mission}} \). In scenario (b) the performance criteria is relaxed, more specifically \( e_{\text{min}} \) and \( e_{\text{max}} \) are extended to \( e_{\text{min}}^{+} \) and \( e_{\text{max}}^{+} \), to achieve the RUL. However, the performance criteria is relaxed by more than what is actually necessary. In scenario (c) the performance criteria is relaxed such that the RUL requirement is satisfied, but not as much as scenario (b).

3.2 Control Architecture

The main elements of the control architecture are depicted in Figure 3 on page 4. The control architecture is comprised of the plant (physical process and production controller), reconfigurable controller and a PHM module. Initially, the production controller is utilized with no modification while the PHM module continuously monitors the system for one (or more) fault mode(s). Once a fault is detected, the RUL is evaluated by the PHM module. If the estimated RUL is greater than the desired RUL, no action is taken. During this period the RUL is re-evaluated periodically. However, if the estimated RUL is less than the desired RUL, a reconfiguration action is triggered. The reconfigurable controller relaxes constraints on the error boundaries by adjusting the weight matrices in the MPC cost function. This continues until either the RUL is satisfied or the weight matrices can no longer be adapted. The remainder of this section presents a detailed description of each module.

3.2.1 Plant (Nominal System)

The plant consists of the production controller and physical process with a control input, \( u \), internal state, \( x \), measured disturbance, \( v \) and output response \( y \). Prognosis based control can only be considered once it’s established the RUL of the plant can be directly controlled and observed. As a result, two important questions arise, “Under what conditions can the RUL of the plant be controlled? ... observed?” These questions are answered by well defined criteria for RUL controllability and RUL observability given in Definitions 7 and 8, accordingly.

**Definition 7** (RUL Controllability). A system is RUL controllable at time \( t_0 \) if there exists a control input, \( u(t) \in \mathcal{U} \) on the interval \( t \in [t_0, t_f] \) such that any initial RUL \( t_{\text{RUL}}(t_0) \) can be driven to any desired RUL value, \( t_{\text{RUL}}(t_f) \in \mathcal{T}_{\text{RUL}} \).

**Definition 8** (RUL Observability). A system is RUL observable at time \( t_0 \) if for any initial state in the state space \( x(t_0) \in \mathcal{X} \) and a given control input \( u(t) \in \mathcal{U} \) defined on the interval \( t \in [t_0, t_f] \) the RUL, \( t_{\text{RUL}} \), can be determined for \( [t_0, t_f] \).

**Remark** 1. If the conditions for RUL controllability and observability are simultaneously satisfied, then the system is said to be RUL stabilizable

**Definition 9** (RUL Stabilizable). A system is RUL stabilizable if for any initial state in the state space \( x(t_0) \in \mathcal{X} \) and any control input \( u(t) \in \mathcal{U} \) defined on the interval \( t \in [t_0, t_f] \) the plant is simultaneously RUL controllable and RUL observable.

3.2.2 Reconfigurable Controller

The two elements of the reconfigurable controller include the low-level supervisor and the MPC controller.

**Low-Level Supervisor** – A logical unit used to continuously monitor the output of the MPC controller to ensure it meets the desired RUL and set-point requirements. More specifically, if the measured RUL, \( t_{\text{RUL}} \), is greater than the desired mission time, \( t_{\text{mission}} \), then no reconfiguration is necessary;
Otherwise new acceptable minimum and maximum allowable tracking errors, $e_{\text{min}}^+$ and $e_{\text{max}}^+$, are adopted. Then, the adjusted set-point, $\Delta u$, and modeled state estimate, $x_m$, are passed to the PHM module to estimate, $t_{\text{RUL}}$. This estimate is used as an input to the adaptation function, $\Gamma$, to update the adaptation parameter, $\rho$. The cost function is updated using a new value for $\rho$ at time-instant $k$. When the estimated RUL, $t_{\text{RUL}}$, is less than the desired mission time, $t_{\text{mission}}$, the adaptation parameter, $\rho$, increases, otherwise it decreases. This process is re-iterated until $\rho \geq \rho_{\text{max}}$. When this occurs, the controller makes no further adaptation attempts. An outline of this process is shown as a flowchart in Figure 4.

**Model Predictive Controller (MPC)** – used to make adjustments to the control signal, $u$, thereby altering the internal states, $x$, and causing the RUL to increase. In the scope of this work, constraints are imposed on the maximum allowable tracking error, $e$. Foreshadowing briefly to the next chapter, it can be proven if $e$ is constrained by $e_{\text{min}} \leq e(t) \leq e_{\text{max}}$ for $\forall t \in [t_0, t_{\text{RUL}}]$, then $\Delta u$ must belong to $U_\delta$,

$$U_\delta \in \{ \Delta u_{\text{min}} \leq \Delta u(t) \leq \Delta u_{\text{max}} | \forall t \in [t_0, t_{\text{RUL}}] \},$$

(10)

where,

$$\begin{align*}
\Delta u_{\text{min}} &= e^+_\text{min} - e^+_{\text{min}}, \\
\Delta u_{\text{max}} &= e^+_\text{max} - e^+_{\text{max}}.
\end{align*}$$

(11)

Now, the optimal set-point adjustment $\Delta u$ is found by minimizing the quadratic cost function,

$$J(x, \Delta u) = \min_{\Delta u \in U_\delta} \left\{ \int_{t_0}^{t_0+T} \left[ (x^* - x)^\top (Q\rho[k]) + (x^* - x) + \Delta u \, R \Delta u \right] dt \right\},$$

(12)

where $x^*$ is the desired state-space value. The weight matrices $Q$ and $R$ are of size $n_x \times n_x$ and $n_r \times n_r$, respectively.

**PHM Module** – In the scope of this work, the PHM module is external to the reconfigurable controller. In general, the prognostic control input to the PHM model includes the reference signal, $r$, modeled state estimate, $x_m$, and set-point adjustment $\Delta u$

### 4 Stability and Uncertainty Analysis

The qualitative overview of the reconfigurable control architecture in the section provides a basis for a quantitative study of set-point reconfiguration with respect to stability and boundedness.

#### 4.1 Reference Model

The MPC requires a reference model of the plant to predict the future set-point adjustments for control reconfiguration. Ideally, the reference model is equivalent to the non-linear plant dynamics. However, using a linear reference model reduces the complexity of the optimal control problem and guarantees a solution exists by optimizing over a convex set. The linear reference model is written in state-space form as,

$$\begin{align*}
\dot{x}_m(t) &= A_m x_m + B_{m,r} r(t) + B_{m,v} v(t), \\
y_m(t) &= C_m x_m + D_{m,v} v(t),
\end{align*}$$

(13)

where $x_m(0) = x_{m0}$ and $A_m, B_{m,r}, B_{m,v}, C_m, D_{m,v}$ are real-valued matrices.

#### 4.2 Composite System

The composite system is comprised of the plant and MPC controller, as shown in Figure 5.

#### 4.2.1 Plant

The control input to the plant, $u$, is defined as,

$$u(t) = r(t) + \Delta u(t),$$

(14)

where $\Delta u$ is a set-point adjustment computed by the MPC. The output of the plant and corresponding tracking error are represented by $y$ and $e$, accordingly.

#### 4.2.2 MPC Controller

The MPC consists of a linear reference model, state observer and an optimizer. The state-observer accepts the current control input, $u$, and plant output, $y$, as inputs. The output of the

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**Figure 4. Flowchart of the low-level supervisor.**
state observer, \( x_m \), is used to initialize the reference model. The reference model is used to predict future state estimates for a given set of future input references over a prediction horizon and takes the form of (13). The optimizer solves for a set-point adjustment \( \Delta u \) by minimizing the cost function in (12).

### 4.3 Error Analysis

Two types of errors are analyzed in this section: tracking error and modeling error. The tracking error corresponds to the difference between the desired input reference, \( r \), and plant output, \( y \). The modeling error represented by the symbol \( e_m \), corresponds to the error in the state estimates that occur as a result of model mismatches propagated over the prediction horizon. The tracking error of the composite system, \( e^+ \) (referred to as the extended tracking error in the previous chapter), is described by the differential equation

\[
\dot{e}^+ (t) = A_r e^+ (t) + \delta_u [k] \cdot \delta (t - kT_s).
\]

where \( A_r \) is Hurwitz and \( e^+ (t_0) = e (t_0) \). The following theorem provides the boundaries for the tracking error of the composite system.

**Theorem 1** (Tracking Error Boundaries with MPC). *Let the tracking error of the composite system be described by (15). If the set-point adjustment, \( \Delta u \), is uniformly bounded in time by \( \Delta u_{\text{min}} \leq \Delta u (t) \leq \Delta u_{\text{max}} \), then the tracking error of the composite system, \( e^+ \), must also be uniformly bounded in time by, \( e (t_0) + \Delta u_{\text{min}} \leq e^+ (t) \leq e (t_0) + \Delta u_{\text{max}} \).

**Proof.** First, the explicit solution of (15) can be found,

\[
e^+ (t) = \exp (A_r (t - t_0)) e (t_0) + \ldots
\]

\[
\int_{t_0}^{t} \exp (A_r (t - \tau)) (\delta_u [k] \cdot \delta (\tau - kT_s)) d\tau.
\]

Applying the translation property of the Dirac-delta function gives,

\[
e^+ (t) = \exp (A_r (t - t_0)) e (t_0) + \ldots
\]

\[
\sum_{n=0}^{k} \delta_u [n] \cdot \exp (A_r (t - nT_s)).
\]

Since \( \Delta u \) is uniformly bounded, the cumulative sum of \( \delta_u \) is bounded by,

\[
\Delta u_{\text{min}} \leq \sum_{n=0}^{k} \delta_u [n] \leq \Delta u_{\text{max}}.
\]

Now, consider the worst case when \( \text{eig} (A_r) \to 0^− \). Under this condition, the explicit expression for \( e^+ \) becomes,

\[
e^+ (t) = e (t_0) + \sum_{n=0}^{k} \delta_u [n].
\]

By applying (18) to (19), the boundary for the case when \( \text{eig} (A_r) \to 0 \) can be given as,

\[
e (t_0) + \Delta u_{\text{min}} \leq e^+ (t) \leq e (t_0) + \Delta u_{\text{max}}.
\]

Finally, if \( A_r \) is Hurwitz, then (17) must always be less than or equal to (19) for all \( t \geq t_0 \). Therefore, by the comparison theorem, (17) must also be bounded by (20).

### 4.4 State-Variable Reconfiguration Analysis

Now that the tracking error of the composite system is shown to be bounded, the effects of set-point adjustment on the future state values can be studied.

#### 4.4.1 Ideal (Matched) Case

First, consider the following definition for the change in the state-variable,

\[
\Delta x (t) \triangleq x^+ (t) - x (t),
\]

where \( x \) is the state of the system if \( \Delta u \equiv 0 \) and \( x^+ \) is the reconfigured state of the system if a non-zero set-point adjustment \( \Delta u \) were applied. Now, consider the case where the linear reference model matches the dynamics of the plant. The predicted change in the state-variable after reconfiguration can be found using Theorem 2.

**Theorem 2** (State Adjustment (Matched Model)). *Consider a closed-loop system which matches the linear-deterministic reference model described by (13). The estimated change in the state, \( \Delta \hat{x} \), at time \( t_{k+q} \) given at time \( t_k \) can be computed by,

\[
\Delta \hat{x} (t_{k+q}) = e^{A_m (t_{k+q} - t_k)} \Delta x_0 + \ldots
\]

\[
\int_{t_k}^{t_{k+q}} \left[ e^{A_m (t-\tau)} \right] B_{m,r} \Delta u (\tau) \]
Proof. The dynamics of the reconfigured state can be expressed as,
\[
\dot{x}^+(t) = A_m x^+(t) + B_{m,r} r(t) + B_{m,r} \Delta u(t) .
\]
Next, taking the time derivative of (21) and substituting the first-order dynamics of \(x\) and \(x^+\) gives,
\[
\Delta \dot{x}(t) = A_m \Delta x(t) + B_{m,r} \Delta u(t) ,
\]
The explicit solution to this first-order differential equation is found as,
\[
\Delta x(t) = e^{A_m(t-t_0)} \Delta x_0 + \int_{t_0}^{t} e^{A_m(t-\tau)} B_{m,r} \Delta u(\tau) \, d\tau .
\]
Finally, since this is assumed to be a perfectly matched model, \(\Delta \dot{x} = \Delta x\). Therefore, the state estimate at time \(t_{k+q}\) given at time \(t_k\) can be found by using (25).

4.4.2 Non-Ideal (Unmatched) Case

The estimated change in the state at time \(t_{k+q}\) given at time \(t_k\) when the reference model does not match the closed-loop system dynamics can be found if the structure of the reference and the closed-loop system models are assumed.

Claim 1 (State Adjustment (Unmatched Model)). Consider the case of the unmatched linear reference model in (13). If modeling error over the prognostic horizon, \(q\), is bounded by a constant \(e_m\) such that,
\[
-|e_m| \leq e_m(t) \leq |e_m| .
\]
for \(\forall t \in [t_k, t_{k+q}]\) at time-instant \(k\), then the change in the state, \(\Delta x\), at time \(t_{k+q}\) given at time \(t_k\) is bounded by,
\[
\Delta x_{lb} \leq \Delta x(t_{k+q}|t_k) \leq \Delta x_{ub} ,
\]
where,
\[
\begin{align*}
\Delta x_{lb} &= \Delta x(t_{k+q}|t_k) - |e_m| \\
\Delta x_{ub} &= \Delta x(t_{k+q}|t_k) + |e_m|
\end{align*}
\]

4.5 RUL Analysis

Given uncertainty boundaries for the state-vector \(x\), the best-case and worst-case prediction boundaries for RUL estimates can be studied in a stochastic manner.

4.5.1 Boundary Conditions

The absolute upper and lower-boundary conditions for each state vector at time \(t\) are defined as \(x_{ub}\) and \(x_{lb}\),
\[
x_{ub} (t_{k+p}|t_k) = \cdot \cdot \cdot
\]
\[
\begin{cases}
x_m + \Delta x_{ub} : |x_m + \Delta x_{ub}| \geq |x_m + \Delta x_{lb}| \\
x_m + \Delta x_{lb} : |x_m + \Delta x_{lb}| < |x_m + \Delta x_{lb}|
\end{cases}
\]

\[
x_{lb} (t_{k+p}|t_k) = \cdot \cdot \cdot
\]
\[
\begin{cases}
x_m + \Delta x_{lb} : |x_m + \Delta x_{lb}| \leq |x_m + \Delta x_{lb}| \\
x_m + \Delta x_{ub} : |x_m + \Delta x_{ub}| > |x_m + \Delta x_{ub}|
\end{cases}
\]

Now, assume that \(\beta_{\Delta x_{lb}} \geq 0\). Then, the lower boundary (or worst case conditions) for RUL must occur when \(x = x_{ub}\),
\[
t^\star_{RUL(lb)} = t_{RUL(lb)}|_{x=x_{ub}} .
\]
Similarly, the upper boundary (or best-case condition) for RUL occurs when \(x = x_{lb}\),
\[
t^\star_{RUL(ub)} = t_{RUL(ub)}|_{x=x_{lb}} .
\]
By applying the lower-bound as the most conservative estimate for \(t_{RUL(lb)}\), the resulting RUL gained after reconfiguration is defined as,
\[
\Delta t_{RUL(lb)} \triangleq t^\star_{RUL(lb)} - t_{RUL(lb)} .
\]
Additionally, the corresponding confidence interval width of the reconfigured RUL is defined as,
\[
c_{RUL} = t^\star_{RUL(ub)} - t^\star_{RUL(lb)} .
\]
An illustration of the predicted fault growth curves for nominal, best-case reconfiguration and worst-case reconfiguration conditions is provided in Figure 6.

4.6 Metrics

Presented are three metrics to evaluate the effectiveness of the reconfiguration routine: remaining life increase (RLI) and prediction uncertainty increase (PUI) and reconfiguration efficiency, represented by the symbol \(\eta\).

4.6.1 Remaining Life Increase (RLI)

RLI is a standardized measure of the relative net increase in RUL, defined as,
\[
RLI \triangleq \frac{t^\star_{RUL(lb)} - t_{RUL(lb)}}{t_{RUL(lb)}} .
\]
For the case when \(RLI < 0\), the RUL decreases (\(\Delta t_{RUL} < 0\)) thereby leading to an implausible or undesirable reconfiguration action.
4.6.2 Prediction Uncertainty Increase (PUI)

PUI is a standardized measure of the relative net increase in the width of the RUL confidence interval, defined as,

$$PUI \triangleq \frac{\epsilon^{\text{RUL}} - \epsilon_{0}}{\epsilon_{0}}.$$  (36)

4.6.3 Reconfiguration Efficiency

Evaluation of RUL feasibility can be difficult to explicitly quantify. A quick estimate of the relative increase in RUL can be made by evaluating the relative change in the cost associated with the plant state before and after reconfiguration. First, define the cost corresponding to the weight $\rho$ as,

$$J(\rho, x_m, \Delta u) = \min_{\Delta u \in U} \left\{ J_x(\rho, x_m) + J_{\Delta u}(\Delta u) \right\},$$  (37)

where,

$$J_x(\rho, x_m) = \rho (x^* - x_m)^T (Q \rho [k]) (x^* - x_m)$$  (38)

and,

$$J_{\Delta u}(\Delta u) = \Delta u^T R \Delta u.$$  (39)

Now, the percent change in the cost before and after reconfiguration, $\eta$, can be computed,

$$\eta(\rho) = \frac{J_x(0, x_m) - J_x(\rho, x_m)}{J_x(0, x_m)}$$  for $\rho \in (0, \infty)$,  (40)

where $\eta > 0$ corresponds to a net increase in RUL and $\eta < 0$ corresponds to a net reduction in RUL.

5 Example Application

An EMA is examined as an example for PHM-based control reconfiguration. An EMA was selected in part due to its availability and its emergence as a solution of choice for future flight control actuation systems. More specifically, the rudder of the NASA X-38 crew re-entry vehicle, shown in Figure 7, was selected as the system of interest. A failure modes, effects and criticality analysis (FMECA) of the X-38 rudder actuator was examined to identify the most critical component, degradation of the motor winding insulation.

5.1 Prognostic Model

In the case of the brushless DC motor, the winding temperature is related to the power loss in the copper windings, assuming the copper losses are the primary source of power loss. A first order thermo-electrical model, shown in Figure 8, can be used to describe the relationship between power loss in the copper windings with respect to the wind-to-ambient temperature (Gokdere et al., 2006; Nestler & Sattler, 1993), represented as $T_{wa}$ and defined as,

$$T_{wa}(t) \triangleq T_w(t) - T_a(t)$$  (41)

where the symbols $T_w$ and $T_a$ correspond to the winding temperature and ambient temperature respectively. The symbols $R_0$, $C_{wa}$ and $R_{wa}$ refer to the winding resistance, thermal capacitance and thermal resistance of the windings, accordingly. The equivalent state space representation can be written as,

$$\dot{T}_{wa} = \frac{1}{R_{wa}C_{wa}} T_{wa} - \frac{R_0}{C_{wa}} i_m^2(t)$$  (42)

Motor winding insulation degrades at a rate related to the winding temperature, $T_w$. Let the RUL be represented as, $t_{RUL}$. The RUL at time $t$ can be related to $T_w$ using Arrhenius’ law (Gokdere et al., 2006),

$$t_{RUL}(t) = c_0 \exp \left( \frac{E_a}{k_B T_w(t)} \right),$$  (43)

where the symbols $E_a$, $k_B$ and $c_0$ are constants representing activation energy, Boltzmann’s constant and an empirical model fit, respectively. Next, let the fault dimension, $L$, be defined as the accumulated RUL consumed,

$$L(t) = L_0 + \int_{t_0}^{t} \frac{1}{t_{RUL}(\tau)} d\tau.$$  (44)

where $L_0$ is the initial fault dimension. Substituting (43) into (44) gives,

$$L(t) = L_0 + \int_{t_0}^{t} c_0^{-1} \exp \left( -\frac{E_a}{k_B T_w(\tau)} \right) d\tau.$$  (45)

By differentiating both sides with respect to time and applying the second fundamental theorem of integral calculus to the right-hand side, an expression for $\dot{L}$ can be found,

$$\dot{L}(t) = c_0^{-1} \exp \left( -\frac{E_a}{k_B T_w(t)} \right),$$  (46)

Figure 8. Schematic of the first-order thermal model.
5.2 Model Uncertainty

Consider the effects of model uncertainty for a linear system with an unmatched linear reference model. In this example only the motor current is of interest; therefore, consideration of the entire state estimate is simplified to the scalar quantity, \( i_m \). Values for \( \Delta A_{m,n} \) and \( \Delta B_{m,r} \) are obtained by using new values for the modeling parameters after adjusting physical modeling parameters randomly to within their corresponding uncertainties. A sinusoidal input with an amplitude of 60 deg was used as the reference applied to the linear actuator model. The sample-time between predictions was set at \( T_s = 0.05 \) s. Monte Carlo simulations were conducted for a range of frequencies from 0.1 Hz to 10 Hz. The percent change in motor current vs. reference frequency was estimated from each set of Monte Carlo simulations. According to the results, the standard deviation of the percent change in motor current is approximately less than 0.114 for 95% of the simulations.

5.3 Long-Term State Predictions with Uncertainty

It can be shown that the modeling error corresponding to a 95% confidence interval is approximately,

\[
-0.395 \hat{i}_m \leq e_m (t) \leq 0.395 \hat{i}_m (t) \tag{47}
\]

where \( \hat{i}_m \) is the estimated value of the motor current. By applying (29) and (30), a 95% confidence interval for the motor current can be expressed as,

\[
0.605 \hat{i}_m (t) \leq i_m (t) \leq 1.395 \hat{i}_m (t) \tag{48}
\]

To demonstrate this boundary, consider the actuator example with the maximum reconfiguration possible, which corresponds to an adaptation parameter \( \rho \gg 1 \). Let the reference signal be sinusoidal with an amplitude of 60 deg and a fixed frequency of 2 Hz. The corresponding actuator position reference signal before and after reconfiguration is shown in Figure 9. Also shown is the set-point adjustment applied to the reference signal and the corresponding motor current with uncertainty boundaries. The reconfiguration efficiency for large values of \( \rho \) was computed as \( \eta = 0.19 \).

5.4 RUL Estimation & Uncertainty

RUL estimation and uncertainty are examined for a simple reference signal. Recall, the input to the prognostic model is the squared current value, \( i_m^2 \), and \( J_x (\rho, x_m) \) is directly proportional to \( i_m^2 \). After reconfiguration, the cost function reduces by a factor of \( \eta \). Therefore, the quantity \( (1 - \eta) i_m^2 \) can be used to represent the input to the prognostic model after reconfiguration. This can be demonstrated using a simple example. Let the reference signal be a sinusoidal input with a frequency of 2 Hz and an amplitude of 60 deg. For these conditions \( i_m \) was simulated as,

\[
i_m (t) = 7.07 \sin (4\pi ft). \tag{49}
\]

For the nominal case when \( \rho = 0 \), the current is bounded by,

\[
4.277 \sin (4\pi t) \leq i_m (t) \leq 9.863 \sin (4\pi t). \tag{50}
\]

and the input to the prognostic model is bounded by,

\[
18.29 \sin^2 (4\pi t) \leq i_m^2 (t) \leq 97.27 \sin^2 (4\pi t). \tag{51}
\]

Similarly, after applying the MPC for \( \rho \gg 1 \), the reconfiguration efficiency becomes \( \eta = 0.19 \). The input to the prognostic model is adjusted by a factor of \( (1 - \eta) \), which becomes,

\[
14.82 \sin^2 (4\pi t) \leq i_m^2 (t) \leq 78.79 \sin^2 (4\pi t). \tag{52}
\]

Applying these boundaries to the input of the prognostic model, a plot of the fault dimension (life consumed) versus the prognostic horizon can be obtained for the cases before and after reconfiguration, provided in Figure 10. From the plot values for \( \Delta \hat{t}_{RUL} \), \( \epsilon_{RUL} \) and \( \epsilon^*_{RUL} \) are computed as
Figure 10. Plot of life consumed versus prognostic horizon before and after reconfiguration for a sinusoidal reference input with a frequency of 2 Hz and an amplitude of 60 deg.

6.178 × 10^3 hrs, 5.393 × 10^3 hrs and 5.003 × 10^3 hrs, accordingly. This allows the metrics RLI and PUI to be computed as 0.116 and −0.080, respectively.

6 Conclusions

This body of work constitutes a significant effort regarding the specific role of RUL in control systems. The overall control scheme was defined as a module which adjusts the reference set-points to the local production controller in order to sacrifice a fixed amount of performance to achieve an increase in RUL. The modules of the reconfigurable controller, the MPC and state observer, were defined mathematically and analyzed to demonstrate stability and boundedness. Finally, the reconfigurable control framework was evaluated using an EMA Simulink model. Results acquired from the simulation demonstrated the feasibility of the approach.

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