

Battery Capacity Estimation of Low-Earth Orbit Satellite Application

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ABSTRACT

Simultaneous estimation of the battery capacity and state-of-charge is a difficult problem because they are dependent on each other and neither is directly measurable. This paper proposes a particle filtering approach for the estimation of the battery state-of-charge and a statistical method to estimate the battery capacity. Two different methods and time scales have been used for this estimation in order to reduce the dependency on each other. The algorithms are validated using experimental data from A123 graphite/LiFePO₄ lithium ion commercial-off-the-shelf cells, aged under partial depth-of-discharge cycling as encountered in low-earth-orbit satellite applications. The model-based method is extensible to battery applications with arbitrary duty-cycles.

1. INTRODUCTION

Health and lifetime uncertainty presents a major barrier to the deployment of lithium-ion (Li-ion) batteries in large-scale aerospace, electric vehicle, and electrical grid applications with stringent life requirements. In the satellite industry, for example, the high cost of launch and the inability to make repairs once in orbit dictate the use of mature battery technologies with conservative duty-cycles to reduce risk. If battery health could be precisely tracked on orbit, the duty-cycle might be tailored to best utilize the remaining life and maximize the value of the investment. Similar opportunities may exist for electric vehicles to maximize battery lifetime by intelligently selecting driving routes and charging strategies. Markets for used electric vehicles and batteries also require accurate battery health assessment to mature to their full potential.

The field of prognostics and health management offers gen-

eral approaches for combining real-time measurements, models and estimation algorithms to track the health and predict the remaining lifetime of batteries (Sheppard, Wilmering, & Kaufman, 2009; Goebel, 2010). Relevant performance/health metrics for battery applications are available power and energy. These can be expressed in terms of battery internal resistance and amp-hour (Ah) capacity, respectively. Battery models are needed to relate capacity and resistance to the current, voltage, and temperature measurement signals available in real-time. For regular predictable duty-cycles such as in unmanned aerial vehicles (Goebel, Saha, Saxena, Celaya, & Christophersen, 2008), simple algebraic relationships between current and voltage may be sufficient. For uncertain duty-cycles such as for electric vehicles, a dynamic model of the current and voltage relationship is necessary. Dynamic models can be in the form of circuit analogs (Verbrugge & Koch, 2006; Plett, 2006), or reduced order physics-based models (Santhanagopalan, Zhang, Kumaresan, & White, 2008; Smith, Rahn, & Wang, 2007; Smith, 2010; J. L. Lee, Chemistruck, & Plett, 2012). Physics-based approaches remain their own active subject of research and thus the simpler circuit model is applied in this work.

State-of-charge (SOC) is usually formulated as a reference model state and can be estimated by using various state estimation methods such as extended Kalman filter (Plett, 2004; J. Lee, Nam, & Cho, 2007; Charkhgard & Farrokhi, 2010; Kim & Cho, 2011; Hu, Youn, & Chung, 2012), unscented Kalman filter (Plett, 2006; Sun, Hu, Zou, & Li, 2011) or cubature Kalman filter (Chen, 2012). Those SOC estimation methods work well in certain situations but would not perform properly in other situations. Extended Kalman filters are prone to linearization errors and both extended Kalman filters and unscented Kalman filters are limited to systems with Gaussian noise distribution. Similar to Kalman filters, particle filters belong to the class of Bayesian estimation methods, but can deal with nonlinear systems with non-Gaussian noise without linearization (Sanjeev Arulampalam, Maskell, Gor-

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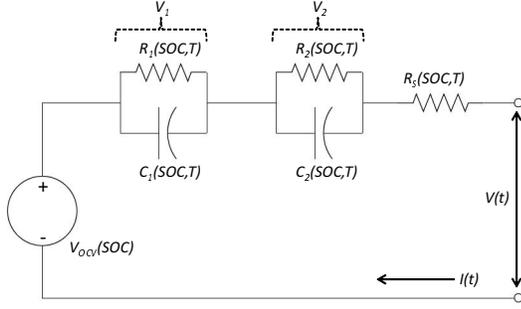


Figure 1. Second order circuit model of a battery

don, & Clapp, 2002). They have been successfully applied to many problems with nonlinear dynamics such as computer vision (Isard & Blake, 1998), speech recognition (Vermaak, Andrieu, Doucet, & Godsill, 2002), robotics (Schulz, Burgard, Fox, & Cremers, 2001), etc. Furthermore, very little work has been done in SOC estimation in conjunction with simultaneous estimation of time-varying battery capacity. This paper proposes a method to estimate both SOC and battery capacity by using a particle filtering approach.

Unlike in the laboratory, in an application environment it is infeasible to completely discharge the battery to obtain a full “ground-truth” measurement of battery total capacity. A key question explored in this paper is to what extent battery total amp-hour (Ah) capacity can be estimated based on only partial discharge data. In addition, estimation of battery capacity using partial discharge data is particularly challenging for Li-ion chemistries with a flat open-circuit voltage relationship versus SOC (Plett, 2011). Such is the case for the Li-ion graphite/iron-phosphate chemistry investigated in the present work.

2. CIRCUIT MODEL

For the reference model, a second-order circuit model is used in this work as shown in Figure 1. While the battery is an infinite-dimensional system, the two time constants of the second order circuit model provide reasonable approximation of voltage/current dynamics for the present application. The state-space equation of this circuit model is expressed as follows:

$$\begin{bmatrix} \dot{SOC}(t) \\ \dot{V}_1(t) \\ \dot{V}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{R_1 C_1} & 0 \\ 0 & 0 & -\frac{1}{R_2 C_2} \end{bmatrix} \begin{bmatrix} SOC(t) \\ V_1(t) \\ V_2(t) \end{bmatrix} + \begin{bmatrix} -\frac{1}{Q} & -\frac{1}{C_1} & -\frac{1}{C_2} \end{bmatrix}^T I(t) + n_1(t) \quad (1)$$

$$V_{out}(t) = V_{ocv}(SOC) - V_1(t) - V_2(t) - R_s \cdot I(t) + v(t). \quad (2)$$

where Q denotes the battery capacity. The values of the parameters R_1 , R_2 , R_s , C_1 and C_2 depend on SOC and time and Q depends on time. (Since the satellite battery considered in this work operates under nearly isothermal conditions, temperature dependency is neglected.)

Measurements of resistance versus SOC exhibit a bathtub shape, with small resistance at mid-SOCs increasing to larger values at low and high SOC extremes. This parametric dependence of R_1 , R_2 and R_s on SOC is captured in Eq. (3)-(5)

$$R_1(SOC(t)) = a_{r1} (1 + b_{r1} \cdot |SOC - c_{r1}|^{d_{r1}}) \quad (3)$$

$$R_2(SOC(t)) = a_{r2} (1 + b_{r2} \cdot |SOC - c_{r2}|^{d_{r2}}) \quad (4)$$

$$R_s(SOC(t)) = a_{rs} (1 + b_{rs} \cdot |SOC - c_{rs}|^{d_{rs}}). \quad (5)$$

As the battery ages, the values of Q slowly decrease and the resistance values slowly increase over time. Since the battery may not be exercised over its entire SOC range in an actual application, only the three relative resistance parameters a_{r1} , a_{r2} and a_{rs} are estimated along with the battery capacity. The dynamics of these time-varying parameters can be formulated by:

$$\begin{bmatrix} Q(k+1) \\ a_{r1}(k+1) \\ a_{r2}(k+1) \\ a_{rs}(k+1) \end{bmatrix} = M \begin{bmatrix} Q(k) \\ a_{r1}(k) \\ a_{r2}(k) \\ a_{rs}(k) \end{bmatrix} + n_2, \quad (6)$$

$$M = \begin{bmatrix} 1 - \varepsilon_1 & 0 & 0 & 0 \\ 0 & 1 + \varepsilon_2 & 0 & 0 \\ 0 & 0 & 1 + \varepsilon_2 & 0 \\ 0 & 0 & 0 & 1 + \varepsilon_2 \end{bmatrix}$$

where ε_1 and ε_2 are small positive constants. We assume n_2 to be constant. We can reformulate a state-space equation by combining Eq.(1) and Eq. (6). Let $x(k) = [SOC(k) \ V_1(k) \ V_2(k) \ Q(k) \ a_{r1}(k) \ a_{r2}(k) \ a_{rs}(k)]^T$ be the augmented state and Δt be the sampling time. Then the discrete-time augmented state-space equation of the second-order circuit model of a battery is expressed as:

$$x(k+1) = Ax(k) + BI(k) + n(k) \quad (7)$$

$$V_{out}(k) = V_{ocv}(SOC) - V_1(k) - V_2(k) - R_s \cdot I(k) + v(k) \quad (8)$$

where

$$A = \text{diag}(1, e^{\lambda_1 \Delta t}, e^{\lambda_2 \Delta t}, M),$$

$$B = \begin{bmatrix} -\Delta t/Q \\ R_1(e^{\lambda_1 \Delta t} - 1) \\ R_2(e^{\lambda_2 \Delta t} - 1) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$n(k) = \text{diag}(n_1(k), n_2).$$

3. PARTICLE FILTER

Particle filtering is a method used to approximate the probability density f_k of the state x_k conditioned on the observations y_0, \dots, y_k ¹. Consider the following nonlinear system:

$$x_k = g(x_{k-1}, u_{k-1}) + n_k \quad (9)$$

$$y_k = h(x_k, u_k) + v_k. \quad (10)$$

where x_k is the state, y_k is the measurement, n_k is the process noise, and v_k is the measurement noise. Suppose that $f_{k-1} = p(x_{k-1} | y_0, \dots, y_{k-1})$ is known. Then the *a priori* distribution of the state x_k can be derived via the Chapman-Kologorov equation²

$$p(x_k | y_0, \dots, y_{k-1}) = \int p(x_k | x_{k-1}) f_{k-1} dx_{k-1} \quad (11)$$

where $p(x_k | x_{k-1})$ represents state transition over time and is determined by the process model (9) and the distribution of the process noise n_k . Note that f_{k-1} is a function of x_{k-1} . This step is called *prediction* or *time propagation*. When the observation y_k at time k is made, the *a priori* distribution is updated using Bayes' rule:

$$\begin{aligned} f_k &= p(x_k | y_0, \dots, y_k) \\ &= \frac{p(y_k | x_k) \cdot p(x_k | y_0, \dots, y_{k-1})}{\int p(y_k | x_k) \cdot p(x_k | y_0, \dots, y_{k-1}) dx_k} \\ &= \frac{p(y_k | x_k) \cdot p(x_k | y_0, \dots, y_{k-1})}{p(y_k | y_0, \dots, y_{k-1})}. \end{aligned} \quad (12)$$

This step is called the *measurement update* as the measurement data y_0, \dots, y_k are used to obtain the *a posteriori* distribution $f_k = p(x_k | y_0, \dots, y_k)$. The distribution $p(y_k | x_k)$ can be obtained from the measurement equations (10) and the distribution of the measurement noise v_k .

Particle filters approximate f_k by a set of weighted samples or particles x_k^i , $i = 1, \dots, N$, where N is the number of particles. For more details about particle filters and sequential Monte Carlo methods, refer to (Sanjeev Arulampalam et al., 2002).

In this paper, sampling importance resampling is used for resampling of the particle filter to reduce degeneracy. The algorithm for the particle filter used in the simulations is given in the following:

1. **Initialization:** $k = 0$
 - Draw x_0^i , $i = 1, \dots, N$ from the initial prior $f_0 = p(x_0)$.
2. For $k = 1, 2, \dots$
 - (a) **Importance Sampling Step**

¹Let the subscript k denote discrete time k for simple notation, i.e., $x_k = x(k)$

² $p(x|y)$ means $p(X = x|Y = y)$ for simplicity of notation where X and Y are random variables and x and y are their realizations.

- **State transition:** For $i = 1, \dots, N$, draw x_k^i from $p(x_k^i | x_{k-1}^i)$, viz., from Eq. (9).
 - **Measurement update and likelihood calculation:** For $i = 1, \dots, N$, evaluate likelihood by calculating $w_k^i = p(y_k | x_k^i)$ after the measurement y_k is available.
 - **Normalization:** Normalize the importance weights $\tilde{w}_k^i = w_k^i / \sum_j w_k^j$.
- (b) **Resampling**
- Resample \hat{x}_k^i using updated weights \tilde{w}_k^i .
 - Set a new weight $w_k^i = 1/N$ for $i = 1, \dots, N$.

4. CAPACITY ESTIMATION

The simultaneous estimation of the battery capacity and SOC is difficult because they are dependent on each other by the relation

$$SOC(t) = SOC(0) - \int_0^t \frac{I(\tau)}{Q} d\tau. \quad (13)$$

Therefore, if the changes in the battery capacity Q are not reflected properly, the calculation of SOC based on Eq. (13) is subject to errors even though the measurement of $I(t)$ is accurate. This paper proposes a novel method to estimate the battery capacity and SOC simultaneously using a particle filter and statistical approach.

The actual value of Q in real situations changes very slowly over time. This paper utilizes past statistical information for an estimate of Q at a longer interval than the sampling time. Let $m \gg 1$ be an integer and $T = m\Delta t$. The battery capacity is estimated at every T and the value of Q in Eq. (1) is set to the estimated battery capacity \hat{Q} at every T other than Δt .

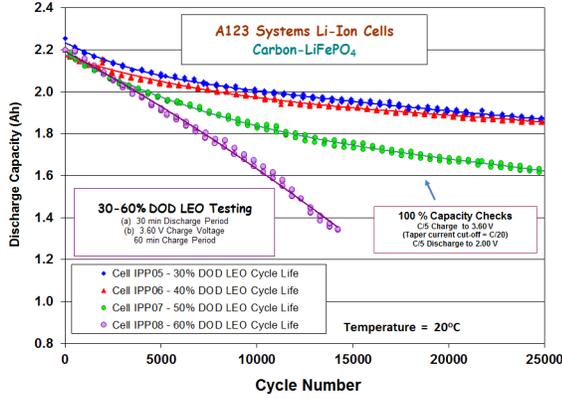
The estimate of x_k by the particle filter is the weighted sample mean of the particles, that is, $\hat{x}_k = \sum_{i=1}^N w_k^i x_k^i$ and the i - j -th element $q_k(i, j)$ of the weighted covariance matrix \mathbf{Q}_k are

$$\begin{aligned} q_k(i, j) &= \frac{\sum_{n=1}^N w_k^n}{\left(\sum_{n=1}^N w_k^n\right)^2 - \sum_{n=1}^N (w_k^n)^2} \times \\ &\sum_{n=1}^N w_k^n (x_k^n(i) - \hat{x}_k(i))(x_k^n(j) - \hat{x}_k(j)) \end{aligned} \quad (14)$$

where $x_k^n(i)$ and $\hat{x}_k(i)$ are the i -th elements of the vectors x_k^n and \hat{x}_k , respectively. The value of $q_k(4, 4)$ implies an estimation error for $x_k(4) = Q(k)$ and the degree of confidence can be represented by the reciprocal of $q_k(4, 4)$. Thus, the paper uses as estimate of Q

$$\hat{Q}(\ell T) = \sum_{k=(\ell-1)T}^{\ell T} \frac{W_k}{\sum_{j=(\ell-1)T}^{\ell T} W_j} \sum_{i=1}^N w_k^i x_k^i(4), \quad \ell = 1, 2, \dots \quad (15)$$

a_{r_s}	b_{r_s}	c_{r_s}	d_{r_s}	a_{r_1}	b_{r_1}	c_{r_1}	d_{r_1}	a_{r_2}	b_{r_2}	c_{r_2}	d_{r_2}
0.0105	112.8616	0.5221	5.5892	0.0440	0.0176	0.6307	3.5007	0.0157	0.0234	0.0029	7.3141

Table 1. Beginning of life parameter values in Eq.(3)-(5) for R_1 , R_2 and R_s Figure 2. Capacity loss during partial DOD cycling of A123 LiFePO₄-based cells (Courtesy Jet Propulsion Laboratory)

where $W_k = 1/q_k(4, 4)$ and the value of $Q(\ell T)$ is reset to a new value of Q in Eq. (1) for every ℓT , $\ell = 1, 2, \dots$. This formulation can be interpreted that W_k is a weight and Eq (15) a weighted time average and re-initialization of state variables.

5. RESULTS

5.1. Low Earth Orbit Satellite Application

For the simulations, we used battery data generated at the Jet Propulsion Laboratory. They performed experiments to evaluate the cycle life performance of A123's 26650 LiFePO₄-based commercial off-the-shelf cells for potential low earth orbit satellite applications. This testing consists of implementing partial depth-of-discharge (DOD) cycling, with 30%, 40%, 50%, and 60% DOD selected. The testing was performed at the room temperature (23°C) and consisted of a 30-minute discharge period and a 60-minute charge period. The charge and discharge rates were scaled proportionately to the corresponding DOD (i.e., the 30% DOD test involved using a 0.4C charge rate and a 0.60C discharge rate). For operational capacity checks (OPCAPS), full charge and discharge of the battery were conducted every 250 cycles. The plots of battery capacity with respect to cycle number are shown in Figure 2. The degradation of battery capacity is clearly observed from the plot.

The analysis contained in this paper focuses upon the 50% DOD data from cycle 2723 to 2815. The battery capacity is reduced to about 2.05 Ah from the initial 2.2 Ah in the range of these cycles.

Least-square regression was used to provide an initial set

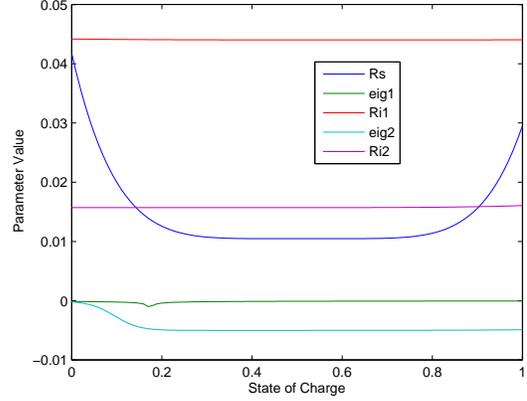


Figure 3. The values of resistances and eigenvalues identified at beginning of life by minimizing the square of voltage error between model and HPPC test data

of parameters representing the battery at beginning of life. Specifically, the Nelder-Mead algorithm (MATLAB function `fminsearch`) minimized the square of the error between the model's output voltage and measured voltage from a hybrid pulse power characterization (HPPC) test run on a cell at beginning of life (Danzer & Hofer, 2008). The beginning-of-life values parameterizing R_1 , R_2 and R_s as functions of SOC are shown in Table 1 and the plots of each resistance and eigenvalue are illustrated in Figure 3. Several nonlinearities arise in the model. Values of open-circuit voltage, $V_{ocv}(SOC)$ in Eq. (2), were taken at 10% increments in SOC following each one-hour rest period of the HPPC test and were implemented in the model as a look-up table. The nonlinearity in Eq. (1) lies in time-varying parameters, R_1 , R_2 , and R_s , which are also dependent on SOC.

The values of the parameters in the particle filter were tuned with simulations. We set the values of ε_1 and ε_2 to be 0.00001. The value of the measurement noise v changes adaptively depending on SOC

$$v(SOC) = \begin{cases} v_0(1 + m_v(0.1 - SOC)), & 0.1 > SOC \\ v_0(1 + m_v(SOC - 0.9)), & 0.9 < SOC \\ v_0, & \text{otherwise} \end{cases}$$

where m_v is a scaling constant depending on measurement error when the value of SOC is very high or low. The process noise n_2 is set to be a constant and n_1 to be a function of SOC

$$n_1(SOC) = \begin{cases} n_{10}, & SOC \geq 0.1 \\ n_{10}(1 + m_w(0.1 - SOC)), & SOC < 0.1 \end{cases}$$

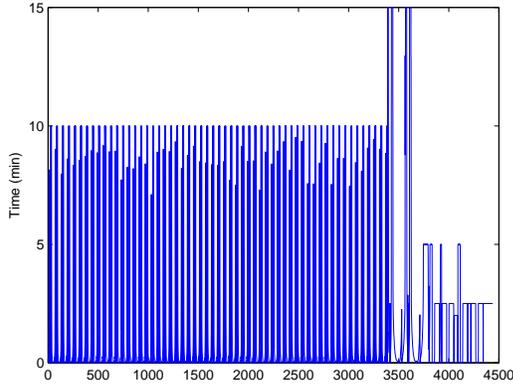


Figure 4. The time intervals between data samples from JPL experiment data

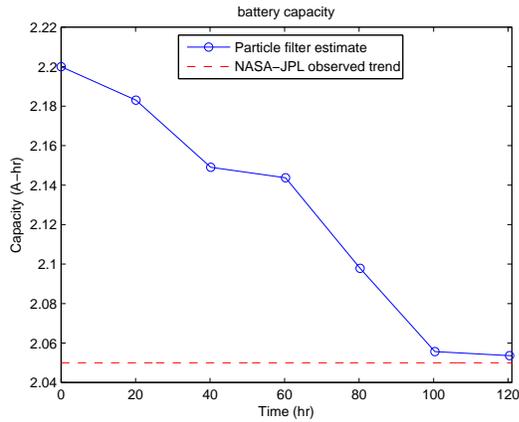


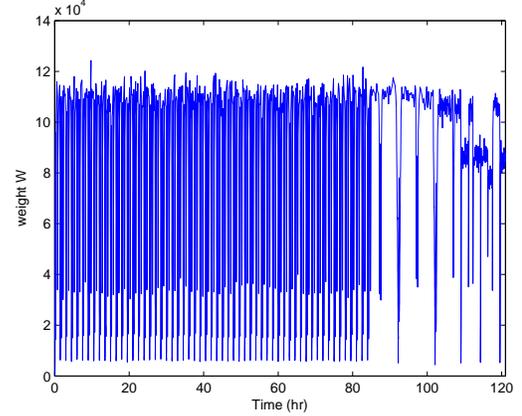
Figure 5. Battery capacity estimation with OPCAPS

where m_w is a scaling constant.

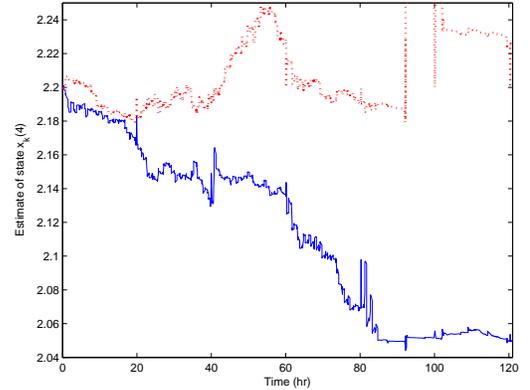
The number of particles used in the simulations is 3,000. The sampling time of the filter is dependent on the interval of measurements. The plot of measurement intervals that are used for the simulations in Section 5.1.1 is shown in Figure 4. Measurements were mostly sampled at every 10 minute and the biggest sampling interval is 15 minute. The particle filter used in the simulations performs stratified resampling (Kitagawa, 1996) if

$$\widehat{N}_{eff} = \frac{1}{\sum_{i=1}^N (w_k^i)^2} < 0.5N$$

where N is the number of particles. Otherwise, the particle filter resamples using the normalized importance weight described in Section 3.



(a) Weight W_k



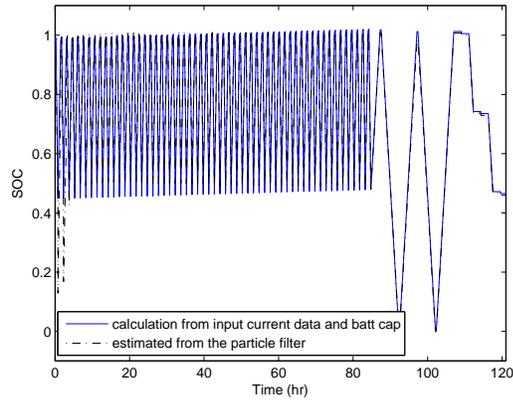
(b) Estimate of $x_k(4)$

Figure 6. Weight W_k and estimate of $x_k(4) = \sum_{i=0}^N w_k^i x_k^i(4)$

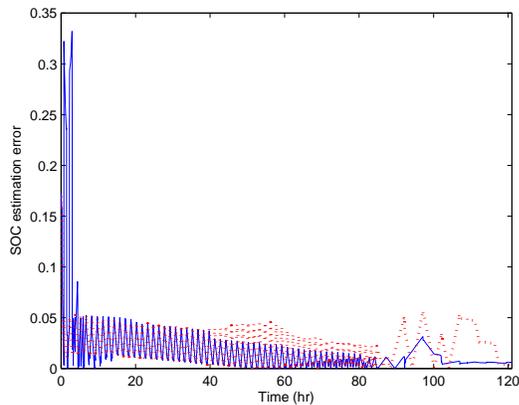
5.1.1. Simulation with Operational Capacity Checks (OPCAPS)

First, we performed simulations with the data from cycle 2773 to 2815 that include OPCAPS. The estimate of the battery capacity was done every 20 hours, that is $T = 20$ hr in Eq. (15). Figure 5 shows the plot of the battery capacity estimate. The initial value of Q is set to 2.2 at time 0, which is the initial battery capacity before battery degradation. This initial value is kept until 20 hr. At $T = 20$ hr, the battery capacity is estimated to be about 2.1831 and the state variable Q in the particle filter is re-initialized to this value, and so on.

The plots of weight $W_k = 1/q_k(4,4)$ and the estimate of $x_k(4) = \sum_{i=0}^N w_k^i x_k^i(4)$ that are used for the battery capacity estimation using Eq. (15) are shown in Figure 6a. Figure 6b presents estimation of state $x_k(4)$ with (solid line) and without (dotted line) the proposed two-time scale method, respectively. It demonstrates that the proposed two-time



(a) SOC estimation



(b) Estimation error

Figure 7. Battery SOC estimation and error with OPCAPS

scale method performs better than particle filtering using augmented state which is usually used for the simultaneous estimation of state and parameters.

The SOC estimate and estimation error are illustrated in Figure 7. The blue solid line in Figure 7a shows the SOC value calculated from input current data and the true battery capacity (2.05 Ah) by using Eq. (13) and the black dash-dotted line illustrates estimated SOC from the particle filter. It can be observed in this plot that the peak value of SOC calculated from Eq. (13) increases over time. This is because measurement errors are accumulated through integration in Eq. (13) and the estimation using the particle filter is more robust to the measurement errors. The estimation error in Figure 7b is the difference between the estimate by the particle filter and the calculated value from input current data and the true battery capacity (2.05 Ah) using Eq. (13). The solid line in Figure 7b indicates SOC estimation error by using the proposed two-time scale method and the dotted line represents error by particle filtering without two-time scale. It shows

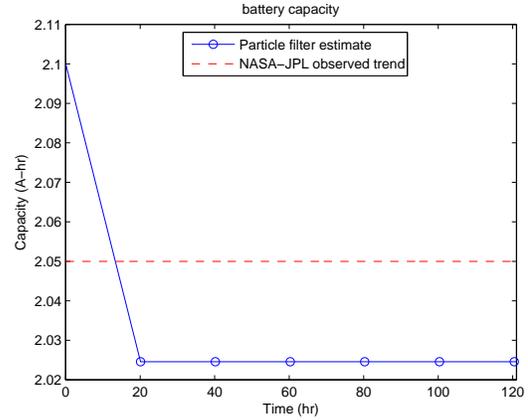


Figure 8. Battery capacity estimation without OPCAPS

that the error goes below 0.01 (1%) after about 100 hr by using the proposed method while the error does not decrease without two-time scale method.

5.1.2. Simulation without Operational Capacity Checks (OPCAPS)

The second simulation was performed with the data from cycle 2723 to 2806, which does not include OPCAPS and only has repeated charge and discharge with 50% DOD. The simulation results are shown in Figure 8, 10 and 9. In this case, the accumulated error in the SOC calculation by Eq. (13) is more noticeable. However, the SOC estimation using a particle filter oscillates between 0.5 and 1, which is the expected SOC range with 50% DOD.

The estimate of the battery capacity converges to about 2.025 Ah, which is a little less than 2.05 Ah, the actual capacity. The errors in SOC and the battery capacity estimation without OPCAPS are greater than those with OPCAPS and it took longer time to converge for SOC estimation. However, the error is about 1.2% and the estimate can be concluded to be accurate even without OPCAPS.

6. CONCLUSION

A method to simultaneously estimate both the capacity and SOC of a Li-ion battery has been proposed using a particle filtering method for SOC estimation and a statistical approach for the battery capacity. The battery capacity estimation has been performed in a different time scale from the SOC estimation and used accumulated past data from both measurement and the particle filter outputs. The estimated value of the battery capacity has been used to modify the parameter of the battery state-space model. Simulation results showed the robust performance of the algorithm in simultaneous estimation with or without operational capacity checks. The proposed method has been shown to perform better than the particle

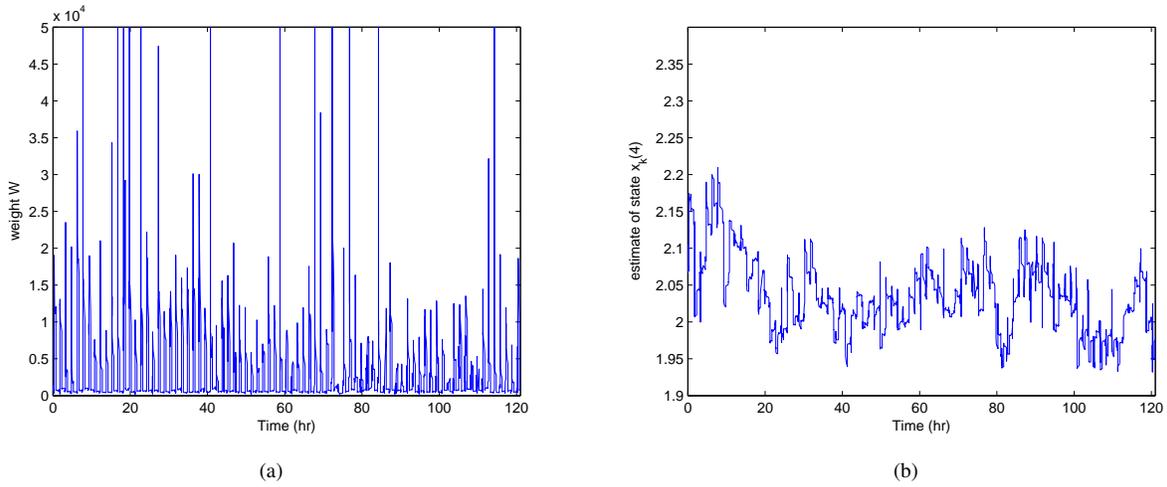


Figure 9. Weight W_k and estimate of $x_k(4) = \sum_{i=0}^N w_k^i x_k^i(4)$

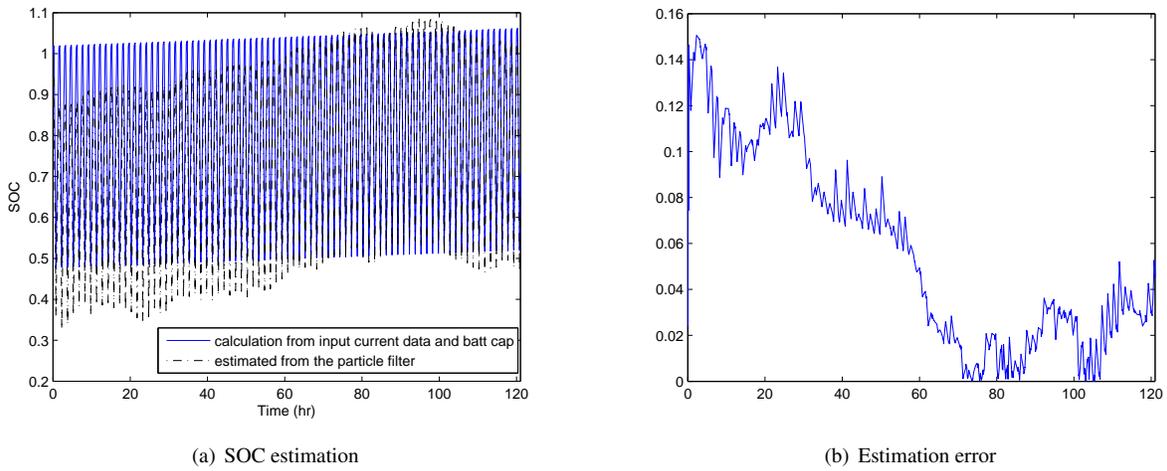


Figure 10. Battery SOC estimation and error without OPCAPS

filter with one time scale and augmented state. Furthermore, accumulation of bias error over time has been shown to be corrected in SOC estimation with the proposed method.

Due to the high cost of launch, satellite batteries are expected to operate until the end of the satellite's life. Unlike the OPCAP used in laboratory tests, in space the battery can never be fully discharged and hence the battery's total capacity must be indirectly estimated. Trending of battery total capacity over lifetime is important for satellite health management to ensure that no regular partial discharge cycle ever exceeds the present capability of the battery, causing loss of the satellite. The proposed method is adequate for the satellite applications since it estimates the battery capacity and SOC robustly even without OPCAPS and measurement errors are not accumulated in SOC estimation unlike Coulomb count method, which indicates that it is suitable for the applications with long operation time.

ACKNOWLEDGMENT

M. Jun, K. Smith and E. Wood gratefully acknowledge the National Renewable Energy Laboratory Laboratory-Directed Research and Development program for battery health algorithm development. The work performed by M. C. Smart was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with National Aeronautics and Space Administration (NASA).

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