

Why is the Remaining Useful Life Prediction Uncertain?

Shankar Sankararaman¹ and Kai Goebel²

¹ *SGT Inc., NASA Ames Research Center, Moffett Field, CA 94035, USA*
shankar.sankararaman@nasa.gov

² *NASA Ames Research Center, Moffett Field, CA 94035, USA*
kai.goebel@nasa.gov

ABSTRACT

This paper discusses the significance and interpretation of uncertainty in the remaining useful life (RUL) prediction of components used in several types of engineering applications, and answers certain fundamental questions such as “Why is the RUL prediction uncertain?”, “How to interpret the uncertainty in the RUL prediction?”, and “How to compute the uncertainty in the RUL prediction?”. Prognostics and RUL prediction are affected by various sources of uncertainty. In order to make meaningful prognostics-based decision-making, it is important to analyze how these sources of uncertainty affect the remaining useful life prediction, and thereby, compute the overall uncertainty in the remaining useful life prediction. The classical (frequentist) and Bayesian (subjective) interpretations of uncertainty and their implications on prognostics are explained, and it is argued that the Bayesian interpretation of uncertainty is more suitable for remaining useful life prediction in the context of condition-based monitoring. Finally, it is demonstrated that the calculation of uncertainty in remaining useful life can be posed as an uncertainty propagation problem, and the practical challenges involved in computing the uncertainty in the remaining useful life prediction are discussed.

1. INTRODUCTION

The prediction of remaining useful life (RUL) is an important functional aspect of an efficient prognostics and health management (PHM) system. The RUL prediction is not only necessary to verify if the mission goal(s) can be accomplished but also important to aid in online decision-making activities such as fault mitigation, mission replanning, etc. Since the prediction of RUL is critical to operations and decision-making, it is imperative that the RUL be estimated accurately.

Since prognostics deals with predicting the future behavior of engineering systems, there are several sources of uncertainty which influence such future prediction, and therefore, it is rarely feasible to obtain an estimate of the RUL with complete precision. In fact, it is not even meaningful to make such predictions without computing the uncertainty associated with RUL. As a result, researchers have been developing different types of approaches for quantifying the uncertainty associated with the RUL prediction and prognostics in general.

Existing methods for quantifying uncertainty in prognostics and remaining useful life prediction can be broadly classified as being applicable to two different types of situations: offline prognostics and online prognostics. Methods for offline prognostics are based on rigorous testing before and/or after operating an engineering system, whereas methods for online prognostics are based on monitoring the performance of the engineering system during operation. For example, there are several research papers which discuss uncertainty quantification in crack growth analysis (Sankararaman, Ling, Shantz, & Mahadevan, 2011; Sankararaman, Ling, & Mahadevan, 2011), structural damage prognosis (Farrar & Lieven, 2007; Coppe, Haftka, Kim, & Yuan, 2010), electronics (Gu, Barker, & Pecht, 2007), and mechanical bearings (Liao, Zhao, & Guo, 2006), primarily in the context of offline testing. Engel et. al (Engel, Gilmartin, Bongort, & Hess, 2000) discuss several issues involved in the estimation of remaining useful life in online prognostics and health monitoring. While some of the initial studies on remaining useful life prediction lacked uncertainty measures (Celaya, Saxena, Kulkarni, Saha, & Goebel, 2012), researchers have recently started investigating the impact of uncertainty on estimating the remaining useful life. For example, there have been several efforts to quantify the uncertainty in remaining useful life of batteries (Saha & Goebel, 2008) and pneumatic valves (Daigle & Goebel, 2010) in the context of online health monitoring. Different types of sampling techniques (Daigle, Saxena, & Goebel, 2012)

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and analytical methods (Sankararaman, Daigle, Saxena, & Goebel, 2013) have been proposed to predict the uncertainty in the remaining useful life.

While the importance of uncertainty quantification in prognostics and RUL estimation have been widely understood, there have been few efforts to understand and appropriately interpret such uncertainty. Celaya et al. (Celaya, Saxena, & Goebel, 2012) discussed the interpretation of RUL in the context of Kalman filtering-based prognostics techniques, and explained that it is not appropriate to arbitrarily force the variance of RUL to be small. It is necessary to further delve into this topic in order to completely analyze the importance and impact of uncertainty in prognostics.

This paper poses three fundamental questions in order to understand uncertainty in prognostics, particular in the context of remaining useful life (RUL) prediction:

1. Why is the RUL prediction uncertain?
2. How do we interpret the uncertainty in RUL?
3. How do we calculate the uncertainty in RUL?

The answers to the above questions are sought from multiple points of view. First, the topic of uncertainty in prognostics is discussed from a qualitative point of view in Section 2; the various sources of uncertainty in prognostics are discussed and the different activities related to uncertainty quantification and management are outlined. Second, the interpretation of uncertainty is discussed from a statistical point of view in Section 3. While statistics and probability methods have been in existence for over 200 years, there has always been a disagreement (amongst mathematicians and statisticians alike) regarding the interpretation of probability. It is important to understand this disagreement before attempting to interpret uncertainty in prognostics. Third, the interpretation of uncertainty in prognostics and RUL prediction is analyzed in detail in Section 4, and it is explained all different interpretations of probability may not be suitable for prognostics and health monitoring purposes. Fourth, it is demonstrated that calculating the uncertainty in RUL is, fundamentally, an uncertainty propagation problem and the challenges in this regard are outlined in Section 5. In this context, it is examined whether it is possible to analytically construct the probability distribution of remaining useful life prediction in certain simple example problems (consisting of linear models and Gaussian variables) and it is demonstrated that it is impossible to estimate closed-form analytical solutions without rigorous mathematical considerations even for such simple example problems.

2. UNCERTAINTY IN PROGNOSTICS

Prognostics is the art of predicting future component/system behavior, identifying possible failure models, and thereby computing the remaining useful life of the compo-

nent/system. There are several sources of uncertainty which affect the prediction of future behavior, and in turn, the remaining useful life prediction. As a result of these sources of uncertainty, it is practically impossible to precisely estimate the remaining useful life prediction. In order to make meaningful prognostics-based decision-making, it is important to analyze how the various sources of uncertainty affect the remaining useful life prediction and compute the overall uncertainty in the remaining useful life prediction.

2.1. Activities Related to Uncertainty in PHM

In the context of prognostics and health management, uncertainties have been discussed from representation, quantification, and management points of view (deNeufville, R., 2004; Hastings, D. and McManus, H., 2004; Ng & Abramson, 1990; Orchard, Kacprzynski, Goebel, Saha, & Vachtsevanos, 2008; Tang, Kacprzynski, Goebel, & Vachtsevanos, 2009). While these three are different processes, they are often confused with each other and interchangeably used. In this paper, the various tasks related to uncertainty quantification and management are classified into four, as explained below. These four tasks need to be performed in order to accurately estimate the uncertainty in the RUL prediction and inform the decision-maker regarding such uncertainty.

1. **Uncertainty Representation and Interpretation:** The first step is uncertainty representation and interpretation, which in many practical applications, is guided by the choice of modeling and simulation frameworks. There are several methods for uncertainty representation that vary in the level of granularity and detail. Some common theories include classical set theory, probability theory, fuzzy set theory, fuzzy measure (plausibility and belief) theory, rough set (upper and lower approximations) theory, etc. Amongst these theories, probability theory has been widely used in the PHM domain (Celaya, Saxena, & Goebel, 2012); even within the context of probabilistic methods, uncertainty can be interpreted and perceived in two different ways: frequentist (classical) versus subjectivist (Bayesian). Sections 3 and 4 outline the differences between these two schools of thought and argues that the Bayesian approach provides a more suitable interpretation for uncertainty in PHM.
2. **Uncertainty Quantification:** The second step is uncertainty quantification, that deals with identifying and characterizing the various sources of uncertainty that may affect prognostics and RUL estimation. It is important that these sources of uncertainty are incorporated into models and simulations as accurately as possible. The common sources of uncertainty in a typical PHM application include modeling errors, model parameters, sensor noise and measurement errors, state estimates (at the time at which prediction needs to be performed), future loading,

operating and environmental conditions, etc. The goal in this step is to address each of these uncertainties separately and quantify them using probabilistic/statistical methods. The Kalman filter is essentially a Bayesian tool for uncertainty quantification, where the uncertainty in the states is estimated continuously as a function of time, based on data which is also typically available continuously as a function of time.

3. **Uncertainty Propagation:** The third step is uncertainty propagation and is most relevant to prognostics, since it accounts for all the previously quantified uncertainties and uses this information to predict (1) future states and the associated uncertainty; and (2) remaining useful life and the associated uncertainty. The former is computed by propagating the various sources of uncertainty through the prediction model. The latter is computed using the estimated uncertainty in the future states along with a Boolean threshold function which is used to indicate end-of-life. In this step, it is important to understand that the future states and remaining useful life predictions are simply dependent upon the various uncertainties characterized in the previous step, and therefore, the distribution type and distribution parameters of future states and remaining useful life should not be arbitrarily chosen. Sometimes, a normal (Gaussian) distribution has been assigned to the remaining useful life prediction; such an assignment is erroneous and the true probability distribution of RUL needs to be estimated through rigorous uncertainty propagation of the various sources of uncertainty through the state space model and the EOL threshold function, both of which may be non-linear in practice.
4. **Uncertainty Management:** The fourth and final step is uncertainty management, and it is unfortunate that, in several articles, the term “Uncertainty Management” has been used instead of uncertainty quantification and/or propagation. Uncertainty management is a general term used to refer to different activities which aid in managing uncertainty in condition-based maintenance during real-time operation. There are several aspects of uncertainty management. One aspect of uncertainty management attempts to answer the question: “Is it possible to improve the uncertainty estimates?” The answer to this question lies in identifying which sources of uncertainty are significant contributors to the uncertainty in the RUL prediction. For example, if the quality of the sensors can be improved, then it may be possible to obtain a better state estimate (with lesser uncertainty) during Kalman filtering, which may in turn lead to a less uncertain RUL prediction. Another aspect of uncertainty management deals with how uncertainty-related information can be used in the decision-making process.

Most of the research in the PHM community pertains to the topics of uncertainty quantification and propagation; few articles have directly addressed the topic of uncertainty management. Even within the realm of uncertainty quantification and propagation, the estimates of uncertainty have sometimes been misinterpreted. For example, when statistical principles are used to estimate a parameter, there is an emphasis on calculating the estimate with the minimum variance. When this principle is applied to RUL estimation, it is important not to arbitrarily reduce the variance of RUL itself. Celaya et al. (Celaya, Saxena, & Goebel, 2012) explored this idea and explained that the variance of RUL needs to be carefully calculated by accounting for the different sources of uncertainty.

2.2. Sources of Uncertainty

In many practical applications, it may even be challenging to identify and quantify the different sources of uncertainty that affect prognostics. Some researchers have classified the different sources of uncertainty into different categories in order to facilitate uncertainty quantification and management. While it has been customary to classify the different sources of uncertainty into aleatory (physical variability) and epistemic (lack of knowledge), such a classification may not be suitable for condition-based monitoring purposes; this point will be explained in detail in the next section. A completely different approach for classification, particularly applicable to condition-based monitoring, is outlined below:

1. **Present uncertainty:** Prior to prognosis, it is important to be able to precisely estimate the condition/state of the component/system at the time at which RUL needs to be computed. This is related to state estimation commonly achieved using filtering. Output data (usually collected through sensors) is used to estimate the state and many filtering approaches are able to provide an estimate of the uncertainty in the state. Practically, it is possible to improve the estimate of the states and thereby reduce the uncertainty, by using better sensors and improved filtering approaches.
2. **Future uncertainty:** The most important source of uncertainty in the context of prognostics is due to the fact that the future is unknown, i.e. both the loading and operating conditions are not known precisely, and it is important to assess the uncertainty in loading and environmental conditions before performing prognostics. If these quantities were known precisely (without any uncertainty), then there would be no uncertainty regarding the *true* remaining useful life of the component/system. However, this true RUL needs to be estimated using a model; the usage of a model imparts additional uncertainty as explained below.
3. **Modeling uncertainty:** It is necessary to use a functional model in order to predict future state behavior.

Further, as mentioned before, the end-of-life is also defined using a Boolean threshold function which indicates end-of-life by checking whether failure has occurred or not. These two models are combinedly used to predict the RUL, and may either be physics-based or data-driven. It may be practically impossible develop models which accurately predict reality. Modeling uncertainty represents the difference between the predicted response and the true response (which can neither be known nor measured accurately), and comprises of several parts: model parameters, model form, and process noise. While it may be possible to quantify these terms until the time of prediction, it is practically challenging to know their values at future time instants.

4. **Prediction method uncertainty:** Even if all the above sources of uncertainty can be quantified accurately, it is necessary to quantify their combined effect on the RUL prediction, and thereby, quantify the overall uncertainty in the RUL prediction. It may not be possible to do this accurately in practice and leads to additional uncertainty. This topic will be revisited again, later in Section 5.

While the different sources of uncertainty and the various uncertainty-related activities have been explained in detail, it is important to understand how to interpret this uncertainty. This topic is detailed in the next section.

3. INTERPRETING UNCERTAINTY

A probabilistic approach to uncertainty representation and quantification has been most commonly used in the prognostics and health management domain. Though probabilistic methods, mathematical axioms and theorems of probability have been well-established in the literature, there is considerable disagreement among researchers on the interpretation of probability. There are two major interpretations based on physical and subjective probabilities, respectively. It is essential to understand the difference between these two interpretations before attempting to interpret the uncertainty in RUL prediction.

3.1. Physical Probability

Physical probabilities (Szabó, 2007), also referred to objective or frequentist probabilities, are related to random physical experiments such as rolling dice, tossing coins, roulette wheels, etc. Each trial of the experiment leads to an event (which is a subset of the sample space), and in the long run of repeated trials, each event tends to occur at a persistent rate, and this rate is referred to as the relative frequency. These relative frequencies are expressed and explained in terms of physical probabilities. Thus, physical probabilities are defined only in the context of random experiments. The theory of classical statistics is based on physical probabilities. Within the realm of physical probabilities, there are two types

of interpretations: von Mises' frequentist (Von Mises, 1981) and Popper's propensity (Popper, 1959); the former is more easily understood and widely used.

In the context of physical probabilities, randomness arises only due to the presence of physical probabilities. If the true value of any particular quantity is deterministic, then it is not possible to associate physical probabilities to that quantity. In other words, when a quantity is not random but unknown, then tools of probability cannot be used to represent this type of uncertainty. For example, the mean of a random variable, sometimes referred to as the population mean, is deterministic. It is meaningless to talk about its probability distribution. In fact, for any type of parameter estimation, the underlying parameter is assumed to be deterministic and only an estimate of this parameter is obtained. The uncertainty in the parameter estimate is addressed through confidence intervals. The interpretation of confidence intervals, as explained in the forthcoming subsection, is sometimes confusing and misleading. Further, the uncertainty in the parameter estimate cannot be used for further uncertainty quantification. For example, if the model parameters of a battery model are estimated under a particular loading condition, then this uncertainty cannot be used for quantifying the battery-response for a similar loading condition. This is a serious limitation, since it is not possible to propagate uncertainty after parameter estimation, which is often necessary in system-level uncertainty quantification (Sankararaman, 2012).

Clearly, there are two limitations of the frequentist interpretation of probability. First, a truly deterministic but unknown quantity cannot be assigned a probability distribution. Second, uncertainty represented using confidence intervals cannot be used for further uncertainty propagation. The second interpretation of probability, i.e. the subjective interpretation, overcomes these limitations.

3.2. Subjective Probability

Subjective probabilities (de Finetti, 1977) can be assigned to any "statement". It is not necessary that the concerned statement is in regard to an event which is a possible outcome of a random experiment. In fact, subjective probabilities can be assigned even in the absence of random experiments. The Bayesian methodology is based on subjective probabilities, which are simply considered to be degrees of belief and quantify the extent to which the "statement" is supported by existing knowledge and available evidence. In recent times, the terms "subjectivist" and "Bayesian" have become synonymous with one another. Calvetti and Somersalo (Calvetti & Somersalo, 2007) explain that "randomness" in the context of physical probabilities is equivalent to "lack of information" in the context of subjective probabilities. In this approach, even deterministic quantities can be represented using probability distributions which reflect the

subjective degree of the analysts belief regarding such quantities. As a result, probability distributions can be assigned to parameters that need to be estimated, and therefore, this interpretation facilitates uncertainty propagation after parameter estimation. Interestingly, subjective probabilities can also be applied in situations where physical probabilities are involved (Sankararaman, 2012).

The concept of likelihood and its use in Bayes' theorem are key to the theory of subjective probability. The numerical implementation of Bayes' theorem may be complicated in some practical cases, and several sampling techniques have been developed by researchers to address this issue. Today, Bayesian methods are used to solve a variety of problems in engineering. Filtering techniques such as particle filtering, Kalman filtering, etc. are also primarily based on the use of Bayes theorem, and sequential sampling.

3.3. Summary

Both the frequentist and subjectivist approaches have been well-established in the literature, in order to aid uncertainty quantification. In fact, both the approaches may yield similar results (but different interpretations) for a few standard problems involving Gaussian variables. Sometimes, both approaches may be suitable for a given problem at hand; for example, Kalman filtering has a purely frequentist interpretation based on least squares minimization as well as a purely Bayesian interpretation which relies on continuously updating the uncertainty in the state estimates using Bayes theorem. It is acceptable to interpret uncertainty using the frequentist approach or the Bayesian approach, provided the interpretation is suitable for the problem at hand. The following section further explores this idea in the context of PHM and RUL estimation.

4. UNDERSTANDING UNCERTAINTY IN RUL

Consider the problem of estimating the remaining useful life prediction, in the context of prognostics and health management. Researchers have pursued two different classes of methods for this purpose; while the first method is based on reliability-testing, the second method is based on condition-monitoring and future behavior prediction. There is a significant difference in the interpretation of uncertainty, when RUL is estimated using these two different approaches. Understanding this difference is important for prognostics and decision-making, and this is focus of the present section.

4.1. Testing-Based Prognostics

Consider a simple numerical example where the remaining useful life needs to be calculated at a given time instant. Assume that a set of run to failure experiments have been performed with high level of control, ensuring same usage and operating conditions. The time to failure for all the n sam-

ples ($r_i; i = 1$ to n) are measured. It is important to understand that *different* RUL values are obtained due to inherent variability across the n different specimens, thereby confirming the presence of physical probabilities. Assume that these random samples belong to an underlying probability density function (PDF) $f_R(r)$, with expected value $E(R) = \mu$ and variance $Var(R) = \sigma^2$. The goal of uncertainty quantification is to characterize this probability density function based on the available n data. Theoretically, an infinite amount of data is necessary to accurately estimate this PDF; however, due to the presence limited data, the estimated PDF is not accurate. As a result, both frequentists and subjectivists express uncertainty regarding the estimate itself. However, frequentists and subjectivists quantify and express this uncertainty in completely different ways.

For the sake of illustration, assume that the entire PDF can be equivalently represented using its mean and variance; in other words, assume that the random variable R follows a two-parameter distribution. Therefore, estimating the parameters μ and σ is equivalent to estimating the PDF. In the context of physical probabilities (frequentist approach), the "true" underlying parameters μ and σ are referred to as "population mean" and "population standard deviation" respectively. Let θ and s denote the mean and the standard deviation of the available n data. As stated earlier, due to the presence of limited data, the sample parameters (θ and s) will not be equal to the corresponding population parameters (μ and σ). The fundamental assumption in this approach is that, since there are true but unknown population parameters, it is meaningless to talk about the probability distribution of any population parameter. Instead, the sample parameters are treated as random variables, i.e., if another set of n data were available, then another realization of θ and s would have been obtained. Using the sample parameters (μ and σ) and the number of data available (n), frequentists construct confidence intervals on the population parameters.

Confidence intervals can be constructed for both μ and σ (Haldar & Mahadevan, 2000). It is important that these intervals be interpreted correctly. As stated earlier, the interpretation of confidence intervals may be confusing and misleading. A 95% confidence interval on μ *does not* imply that "the probability that μ lies in the interval is equal to 95%"; such a statement is wrong because μ is purely deterministic and physical probabilities cannot be associated with it. The random variable here is in fact θ , and the interval calculated using θ . Therefore, the correct implication is that "the probability that the estimated confidence interval contains the true population mean is equal to 95%".

Alternatively, it is also possible to address the problem of computing $f_R(r)$ purely from a subjective (Bayesian) point of view. One important difference now is that the Bayesian approach does not clearly differentiate between "sample param-

eters” and “population parameters”. The probability distribution of μ is directly computed using the available data (recall that this was impossible in the frequentist approach since μ is the true parameter and precise but unknown), and this uncertainty is referred to as the analyst’s degree of belief for the underlying true parameter μ . Similarly, the probability distribution of σ can also be computed. Recall that one realization of the parameters (μ and σ) uniquely define the PDF $f_R(r)$. However, since the parameters are themselves uncertain, R is now represented by a family of distributions (Sankararaman & Mahadevan, 2011), reflective of the fact that there is limited data. This family of distributions will shrink to the true underlying PDF as the number of available data increases.

4.2. Condition-Based Prognostics

Most of the discussion pertaining to testing-based prognostics is not applicable to condition-based monitoring and prognostics. The distinctive feature of condition-based monitoring is that each component/subsystem/system is considered by itself, and therefore, “variability across specimens” is non-existent. Any such “variability” is spurious and must not be considered. At any generic time instant t_P at which prognostics needs to be performed, the component/subsystem/system is at a specific state. The actual state of the system is purely deterministic, i.e., the true value is completely precise, however unknown. Therefore, if a probability distribution is assigned for this state, then this distribution is simply reflective of the analyst’s knowledge regarding this state and cannot be interpreted from a frequentist point of view. Thus, by virtue of definition of condition-based monitoring, physical probabilities are not present here, and a subjective (Bayesian) approach is only suitable for uncertainty quantification.

The goal in condition-based prognostics is, at any generic time instant t_P , to predict the remaining useful life of the component/subsystem/system as condition-based estimate of the usage time left until failure. First, measurements until time t_P are used to estimate the state at time t_P . Then, using a forecasting method (which may be model-based or data-driven), future state values (corresponding to time instants greater than t_P) are computed. In order to forecast future state values, it is also necessary to assume future loading conditions (and operating conditions) which is a major challenge in condition-based prognostics. Typically, the analyst subjectively assumes statistics for future loading conditions based on past experience and existing knowledge; thus, the subjective interpretation of uncertainty is clearly consistent across the entire condition-based monitoring procedure, and therefore, inferences made out of condition-based monitoring also need to be interpreted subjectively. This forecasting is stopped when failure is reached, as indicated by the aforementioned boolean threshold function. This indicates the end-of-life (EOL) and the EOL can be directly used to compute the remaining useful life (RUL) prediction. Note

that it is important to interpret the uncertainty in EOL and RUL subjectively.

4.3. Why is the RUL Prediction Uncertain?

In light of the above discussion, it is necessary to revisit the question “Why is the RUL uncertain?” from a new perspective. While Section 2 explained that RUL is uncertain because there are several sources of uncertainty which influence RUL estimation, now it is clear that the uncertainty in RUL could arise due to variability across multiple specimens (testing-based prognostics scenario) or simply due to subjective uncertainty regarding a single specimen (condition-based prognostics). The following section discusses the computation of RUL in detail by presenting a detailed framework for uncertainty quantification in prognostics, and explains how to calculate the uncertainty in remaining useful life prediction.

5. UNCERTAINTY QUANTIFICATION IN RUL

First, a general computational framework for uncertainty quantification in prognostics and remaining useful life prediction is presented. Second, it is illustrated as to how the problem of computing uncertainty in the remaining useful life prediction can be viewed as an uncertainty propagation problem. Third, the need of rigorous mathematical algorithms for uncertainty quantification in RUL is demonstrated using certain numerical examples. Finally, the challenges involved in computing RUL uncertainty are discussed in detail.

5.1. Computational Framework for Prognostics

Suppose that it is desired to perform prognostics and predict the RUL at a generic time-instant t_P . Daigle and Goebel (Daigle & Goebel, 2011) explain that it is important to develop an architecture for model-based prognostics for practical engineering purposes. This paper considers the architecture in Fig. 1, where the whole problem of prognostics can be considered to consist of the following three sub-problems:

1. Present state estimation
2. Future state prediction
3. RUL computation

5.1.1. State Estimation

The first step of estimating the state at t_P serves as the precursor to prognosis and RUL computation. Consider the state space model which is used to continuously predict the state of the system, as:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{u}(t), \mathbf{v}(t)) \quad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^{n_x}$ is the state vector, $\boldsymbol{\theta}(t) \in \mathbb{R}^{n_\theta}$ is the parameter vector, $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ is the input vector, $\mathbf{v}(t) \in \mathbb{R}^{n_v}$ is the process noise vector, and \mathbf{f} is the state equation.

The state vector at time t_P , i.e., $\mathbf{x}(t)$ (and the parameters $\boldsymbol{\theta}(t)$,

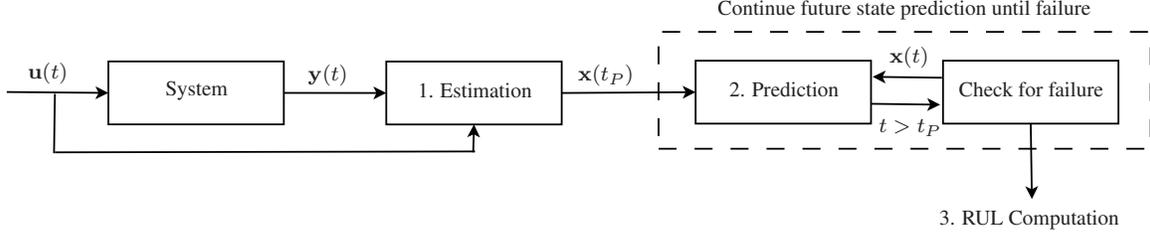


Figure 1. Model-Based Prognostics Architecture

if they are unknown) is (are) estimated using output data collected until t_P . Let $\mathbf{y}(t) \in \mathbb{R}^{n_y}$, $\mathbf{n}(t) \in \mathbb{R}^{n_n}$, and \mathbf{h} denote the output vector, measurement noise vector, and output equation respectively. Then,

$$\mathbf{y}(t) = \mathbf{h}(t, \mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{u}(t), \mathbf{n}(t)) \quad (2)$$

Typically, filtering approaches such as Kalman filtering, particle filtering, etc. may be used for such state estimation. It must be recalled that these filtering methods are collectively known as Bayesian tracking methods, not only because they use Bayes theorem for state estimation but also are based on the subjective interpretation of uncertainty. In other words, any time instant, there is nothing uncertain regarding the true states. However, the true states are not known precisely, and therefore, the probability distributions of these state variables are estimated through filtering. The estimated probability distributions are simply reflective of the subjective knowledge regarding those state variables.

5.1.2. State Prediction

Having estimated the state at time t_P , Eq. 1 is used to predict the future states of the component/system. This differential equation can be discretized and used to predict $\mathbf{x}(t+1)$ as a function of $\mathbf{x}(t)$. Therefore, using this recursive relation, the state at any future time instant $t > t_P$ can be calculated.

5.1.3. RUL Computation

RUL computation is concerned with the performance of the component that lies outside a given region of acceptable behavior. The desired performance is expressed through a set of n_c constraints, $C_{EOL} = \{c_i\}_{i=1}^{n_c}$, where $c_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\theta} \times \mathbb{R}^{n_u} \rightarrow \mathbb{B}$ maps a given point in the joint state-parameter space given the current inputs, $(\mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{u}(t))$, to the Boolean domain $\mathbb{B} \triangleq [0, 1]$, where $c_i(\mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{u}(t)) = 1$ if the constraint is satisfied, and 0 otherwise (Daigle & Goebel, 2013).

These individual constraints may be combined into a single *threshold function* $T_{EOL} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\theta} \times \mathbb{R}^{n_u} \rightarrow \mathbb{B}$, defined as:

$$T_{EOL}(\mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{u}(t)) = \begin{cases} 1, & 0 \in \{c_i(\mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{u}(t))\}_{i=1}^{n_c} \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

T_{EOL} is equal to 1 when any of the constraints are violated. Then, the End of Life (EOL, denoted by E) at any time instant t_P is then defined as the earliest time point at which the value of T_{EOL} becomes equal to one. Therefore,

$$E(t_P) \triangleq \inf\{t \in \mathbb{R} : t \geq t_P \wedge T_{EOL}(\mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{u}(t)) = 1\}. \quad (4)$$

The Remaining Useful Life (RUL, denoted by R) at time instant t_P is expressed as:

$$R(t_P) \triangleq E(t_P) - t_P. \quad (5)$$

Note that the output equation (Eq. 2) or output data ($\mathbf{y}(t)$) is not used in the prediction stage, and EOL and RUL are dependent only on the state estimates at time t_P ; though these state estimates are obtained using the output data, the output data is not used for EOL/RUL calculation after state estimation.

For the purpose of implementation, \mathbf{f} in Eq. 1 is transformed into the corresponding discrete-time version.

5.2. RUL Estimation through Uncertainty Propagation

Thus, it is clear that RUL predicted at time t_P , i.e., $R(t_P)$ depends on

1. Present state estimate ($\mathbf{x}(t_P)$); using the present state estimate and the state space equations in Eq. 1, the future states ($\mathbf{x}(t_P), \mathbf{x}(t_P+1), \mathbf{x}(t_P+2), \dots, \mathbf{x}(t_P+R(t_P))$) can be calculated.
2. Future loading ($\mathbf{u}(t_P), \mathbf{u}(t_P+1), \mathbf{u}(t_P+2), \dots, \mathbf{u}(t_P+R(t_P))$); these values are needed to calculate the future state values using the state space equations.
3. Parameter values from time t_P until time $t_P+R(t_P)$ (denoted by $\boldsymbol{\theta}(t_P), \boldsymbol{\theta}(t_P+1), \dots, \boldsymbol{\theta}(t_P+R(t_P))$).
4. Process noise ($\mathbf{v}(t_P), \mathbf{v}(t_P+1), \mathbf{v}(t_P+2), \dots, \mathbf{v}(t_P+R(t_P))$).

For the purpose of RUL prediction, all of the above quantities are independent quantities and hence, RUL becomes a dependent quantity. Let $\mathbf{X} = \{X_1, X_2, \dots, X_i, \dots, X_n\}$ denote the vector of all the above dependent quantities, where n is the length of the vector \mathbf{X} , and therefore the number of uncertain quantities that influence the RUL prediction. Then the calculation of RUL (denoted by R) can be expressed in terms

of a function, as:

$$R = G(\mathbf{X}) \quad (6)$$

The above functional relation in Eq. 6 can be graphically explained, as shown in Fig. 2.

For example, consider the case where the component/system is subjected to uniform loading (characterized by one variable, the amplitude which remains constant with time), modeled using one parameter (which is time-invariant), and characterized using two states (the state estimates at time t_P and Eq. 1 can be used to predict the state values at any future time instant). Then, excluding the effect of process noise, there are $n = 4$ quantities that affect the RUL prediction. Note that there are $R(t_P) + 1$ process noise terms for each state; therefore, the inclusion of process noise increases the value of n , and therefore the dimensionality of the problem. This raises a practical concern and has been addressed in an earlier publication by replacing the time-variant process noise using an equivalent time-invariant process noise (Sankararaman & Goebel, 2013). In the rest of the paper, a generalized framework is presented without using this equivalent time-invariant process noise concept.

Knowing the values of \mathbf{X} , it is possible to compute the value of R , using Fig. 2 that is equivalently represented by Eq. 6. The quantities contained in \mathbf{X} are uncertain, and the focus in prognostics to compute their combined effect on the RUL prediction, and thereby compute the probability distribution of R . The problem of estimating the uncertainty in R is equivalent to propagating the uncertainty in \mathbf{X} through G , and it is necessary to use computational methods for this purpose.

5.3. Need for Computational Approaches

The problem of estimating the uncertainty in R using uncertainty propagation techniques is a non-trivial problem, and needs rigorous computational approaches. This involves estimating the probability density function of R (PDF, denoted by $f_R(r)$) or equivalently the cumulative distribution function of R (CDF, denoted by $F_R(r)$). In some rare cases, it is possible to analytically obtain the distribution of R . Some of such special cases are listed below:

1. Each and every quantity contained in \mathbf{X} follows a normal (Gaussian) distribution, and the function G can be expressed as a weighted linear combination of the quantities in \mathbf{X} . In this case, R also follows a normal distribution, and its statistics can be calculated analytically.
2. Each and every quantity contained in \mathbf{X} follows a lognormal distribution, and if the *logarithm* of the function G can be expressed as a weighted combination of the quantities in \mathbf{X} , then $\log(R)$ follows a normal distribution whose statistics can be estimated analytically. In other words, R also follows a lognormal distribution.

While Gaussian distributions and linear state space models (linear \mathbf{f} in Eq. 1) may be commonly used in the prognostics and health management domain, it is important to understand that using linear state space models is not equivalent to G being linear. In other words, the use of the threshold function along with the linear state models automatically renders G non-linear.

In order to illustrate this important point, and to emphasize the importance of using rigorous computational methods, consider a simple example where the state equation is given by:

$$x(t+1) = a.x(t) + b. \quad (7)$$

Assume that a suitable time-discretization has been chosen for the purpose of implementation. It is desired to predict future behavior and compute RUL at $t_P = 0$, and state value at this time is denoted by $x(0)$ which is a Gaussian random variable. Further a and b are constants (i.e., not random) which are used to predict future states. It can be easily demonstrated that the state value at any future time instant can be expressed as a function of $x(0)$.

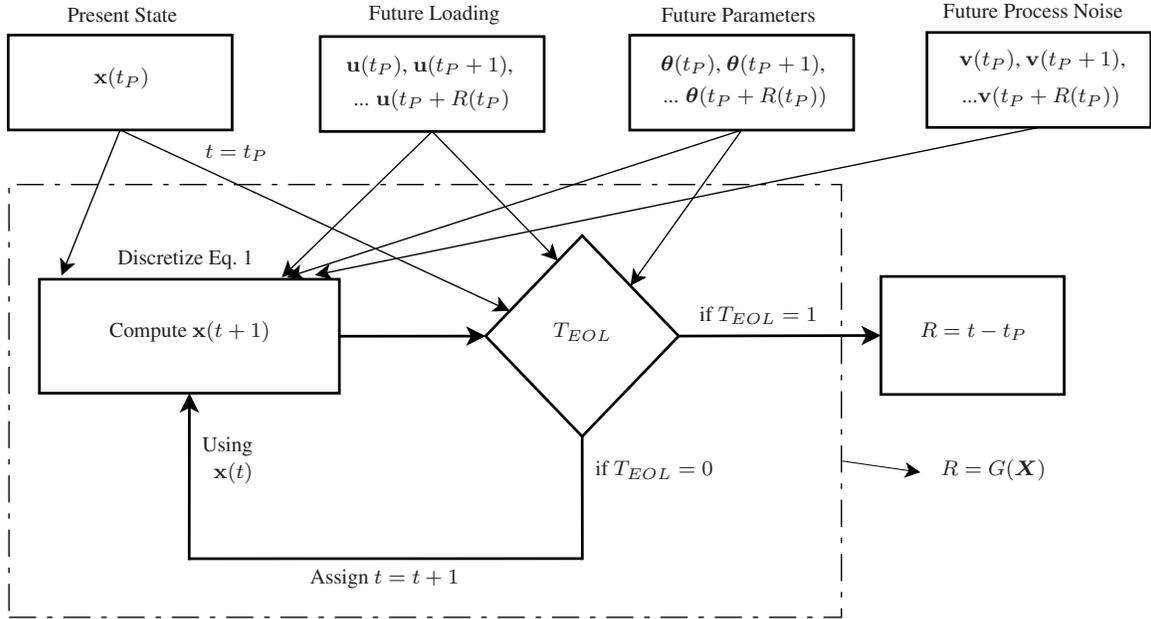
$$x(n) = a^n.x(0) + \sum_{j=0}^{j=n-1} a^j b \quad (8)$$

It is clear from Eq. 8 that the state value at any future time instant is a linear function of $x(0)$, and therefore is also Gaussian. In order to compute the remaining useful life, it is necessary to choose a threshold function. Depending on the choices of a and b , $x(n)$ may either be an increasing function or a decreasing function. If $x(n)$ is a decreasing function, then the threshold function will indicate that failure occurs when the state value x becomes smaller than a critical lower bound (l). Alternatively, if $x(n)$ is an increasing function, then the threshold function will indicate that failure occurs when the state value x becomes greater than a critical upper bound u . Without loss of generality, any of the two cases may be chosen for illustrative purposes. For example, consider that $x(n)$ is decreasing and failure happens when $x < l$. Therefore, the remaining useful life (r , an instance of the random variable R) is equal to the smallest n such that $x(n) < l$. Therefore RUL can be calculated as

$$r = \inf\{n : a^n.x(0) + \sum_{j=0}^{j=n-1} a^j b < l\}, \quad (9)$$

Assuming that the chosen time-discretization level is infinitesimally small, it is possible to directly estimate the RUL by solving the equation:

$$a^r.x(0) + \sum_{j=0}^{j=r-1} a^j.b = l. \quad (10)$$

Figure 2. Definition of G

The above equation calculates the RUL (r) as a function of the initial state ($x(0)$). Hence, the above equation is similar to G defined earlier in Fig. 2. The difference now is that the only considered source of uncertainty is the state estimate $x(0)$; model uncertainty, future loading uncertainty, etc. are not considered here. The RUL R follows a Gaussian distribution if and only if it is linearly dependent on $x(0)$. In other words, R follows a Gaussian distribution if and only if Eq. 10 can be rewritten as:

$$\alpha.r + \beta.x(0) + \gamma = 0 \quad (11)$$

for some arbitrary values of α , β , and γ . If it were possible to estimate such values for α , β , and γ , the distribution of RUL can be obtained analytically.

In order to examine if this is possible, rewrite Eq. 10 as:

$$x(0) = \frac{1}{a^r} \left(l - \sum_{j=0}^{j=r-1} a^j \cdot b \right) \quad (12)$$

While $x(0)$ is completely on the left hand side of this equation, r appears not only as an exponent in the denominator but is also indicative of the number of terms in the summation on the right hand side of the above equation. Therefore, it is clear that the relationship between r and $x(0)$ is not linear. Therefore, even if the initial state ($x(0)$, a realization of $X(0)$) follows a Gaussian distribution, the RUL (r , a realization of R) does not follow a Gaussian distribution. Thus, it is clear that even for a simple problem consisting of linear state models, an extremely simple threshold function, and only one uncertain variable that is Gaussian, the calculation of the probability distribution of R is neither trivial nor straight-

forward.

Practical problems in the prognostics and health management domain may consist of:

1. Several non-Gaussian random variables which affect the RUL prediction,
2. A non-linear multi-dimensional state space model,
3. Uncertain future loading conditions,
4. A complicated threshold function which may be defined in multi-dimensional space.

The fact that the distribution of RUL simply depends on the quantities indicated in Fig. 2 implies that it is technically inaccurate to artificially assign the probability distribution type (or any statistic such as the mean or variance) to RUL.

5.4. Illustrations

Sometimes, the probability distribution of RUL may be extremely skewed; for example, the RUL of a lithium-ion battery used to power an unmanned aerial vehicle was predicted by Sankararaman et al. (Sankararaman et al., 2013) and it was observed that the probability distribution was particularly skewed near failure. The PDFs of the End-of-Discharge (EOD) prediction at various time instants ($T = 0$ seconds through $T = 4000$) are shown in Fig. 3 and then the PDF of End-of-Discharge predicted at $T = 5000$ seconds (which corresponds to near-failure) is indicated in Fig. 4. The RUL can be calculated by simply subtracting the prediction time-instant from the EOD prediction. Note the significant change in the shape of the PDF near failure. It is extremely important to be able to accurately predict the RUL particularly as failure

is approaching, and it is clear from Fig. 4 that assuming a normal distribution or an arbitrary standard deviation would not be able to achieve this goal; only a theoretically accurate uncertainty quantification method can reproduce this probability distribution, whose mode almost coincides with its lower bound (left-hand-side tail).

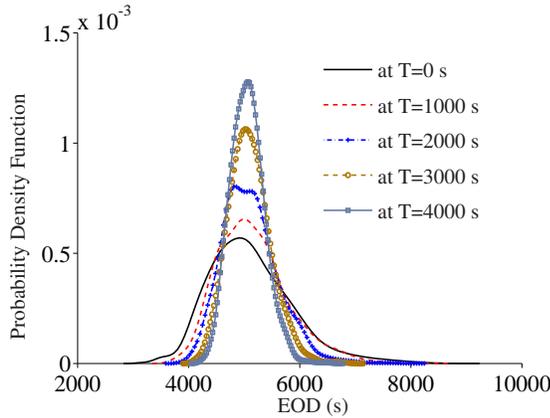


Figure 3. EOD Prediction at Multiple Time Instants

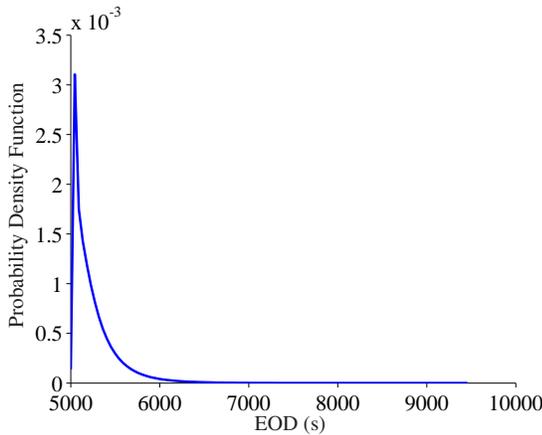


Figure 4. EOD Prediction at $T = 5000$ seconds (near failure)

Sometimes, depending on the chosen statistics of future loading conditions, the distribution of EOD may even be multi-modal. For example, Saha et al. (Saha & Goebel, 2008) calculated future loading statistics that lead to a multi-modal PDF for the EOD, as shown in Fig. 5.

It is important to capture such characteristics of the RUL (which is equivalent to the end-of-discharge in Fig 3-5) probability distribution, and this can be accomplished only by using accurate uncertainty quantification methodologies without making critical assumptions regarding the shape of the PDF of the RUL, its mean, median, mode, standard deviation, etc. Therefore, the goal must be to accurately calculate the probability distribution of R by propagating the different sources of uncertainty through G as indicated in Fig. 2.

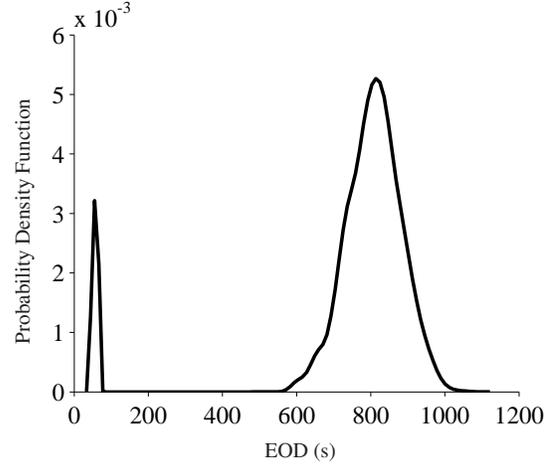


Figure 5. A Multi-Modal PDF

5.5. Uncertainty Propagation Methods

In order to answer the obvious question: “How to calculate the uncertainty in R and estimate the PDF of R ?”, it is necessary to resort to rigorous computational methodologies which have been developed by statisticians and researchers in the field of uncertainty quantification in order to solve a typical uncertainty propagation problem. There are different types of sampling methods such as Monte Carlo sampling (Caflich, 1998), Latin hypercube sampling (Loh, 1996), adaptive sampling (Bucher, 1988), importance sampling (Glynn & Iglehart, 1989), unscented transform sampling (Van Zandt, 2001), etc. Alternatively, there are analytical methods such as the first-order second moment method (Dolinski, 1983), first-order reliability method (Hohenbichler & Rackwitz, 1983), second-order reliability method (Der Kiureghian, Lin, & Hwang, 1987), etc. In addition, there are also methods such as the efficient global reliability analysis (Bichon, Eldred, Swiler, Mahadevan, & McFarland, 2008) method which involve both sampling and the use of analytical techniques. All of these methods empirically calculate the probability distribution of RUL; while some of these methods calculate the PDF ($f_R(r)$) of RUL, some other methods calculate the CDF ($F_R(r)$), and some other methods directly generate samples from the target probability density function ($f_R(r)$). Due to some limitations of each of these methods, it may not be possible to accurately calculate the actual probability distribution of R . Accurate calculation is possible only by using infinite samples for Monte Carlo sampling. Any other method (for example, the use of a limited, finite number of samples) will lead to uncertainty in the estimated probability distribution, and this additional uncertainty is referred to as prediction-method uncertainty. It is possible to decrease (and maybe eventually eliminate) this type of uncertainty either by using advanced probability techniques or powerful computing power.

It is necessary to further investigate the aforementioned un-

certainty propagation methods, and identify whether they can be applied to prognostics health monitoring. Some earlier publications have investigated the use of certain methods such as Monte Carlo sampling, unscented transform sampling, first-order reliability methods, etc. in this regard.

5.6. Challenges

There are several challenges in using different uncertainty quantification methods for prognostics, health management and decision-making. It is not only important to understand these challenges but also necessary to understand the requirements of PHM systems in order to integrate efficient uncertainty quantification along with prognostics and aid risk-informed decision-making. Some of the issues involved in such integration are outlined below:

1. An uncertainty quantification methodology for prognostics needs to be computationally feasible for implementation in online health monitoring. This requires quick calculations, while uncertainty quantification methods have been traditionally known to be time-consuming and computationally intensive.
2. Sometimes, the probability distribution of RUL may be multi-modal and the uncertainty quantification methodology needs to be able to accurately capture such distributions.
3. Existing verification, validation, and certification protocols require algorithms to produce deterministic, i.e., repeatable calculations. Several uncertainty quantification methods are non-deterministic, i.e. produce different (albeit, only slightly if implemented well) results on repetition.
4. The uncertainty quantification method needs to be accurate, i.e., the entire probability distribution of \mathbf{X} needs to be correctly accounted for, and the functional relationship defined by G in Fig. 2. Some methods use only a few statistics (usually, mean and variance) of \mathbf{X} and some methods make approximations (say for example, linear) of G . Finally, it is important to correctly propagate the uncertainty to compute the entire probability distribution of RUL.
5. While it is important to be able to calculate the entire probability distribution of RUL, it is also important to be able to quickly obtain bounds on RUL which can be useful for online decision-making.

Each uncertainty quantification method may address one or more of the above issues, and therefore, it may even be necessary to resort to different methods to achieve different goals. Future research needs to continue this investigation, analyze different types of uncertainty quantification methods and study their applicability to prognostics before these methods can be applied in practice.

6. CONCLUSION

This paper discussed the significance and interpretation of uncertainty the context of prognostics and health management. The prediction of remaining useful life in engineering systems is affected by several sources of uncertainty, and it is important to correctly interpret this uncertainty in order to facilitate meaningful decision-making. Uncertainty can be interpreted in two ways, either in terms of physical probabilities from a frequentist point of view or in terms of subjective probabilities from a Bayesian point of view. While a frequentist interpretation may be suitable for testing-based prognostics, there are no physical probabilities in the context of condition-based prognostics. Therefore, uncertainty in the context of condition-based monitoring needs to be interpreted subjectively, and hence, a Bayesian approach is more suitable for this purpose. It was also explained that Bayesian tracking methods for state estimation are so-called not only because they use Bayes theorem but are also based on the principle of subjective probability.

This paper also emphasized the importance of accurately computing the uncertainty in the remaining useful life prediction. It was illustrated that it may not be analytically possible to calculate the uncertainty in the remaining useful life prediction even for certain simple problems involving Gaussian random variables and linear state-prediction models. Therefore, it is necessary to resort to computational methodologies for such uncertainty quantification and compute the probability distribution of remaining useful life prediction. In this process, it is important not to make assumptions regarding the shape of the probability distribution of the remaining useful life prediction or any of its statistics such as the mean, median, standard deviation, etc.

Finally, it was explained that the problem of estimating the probability distribution of remaining useful life can be viewed as an uncertainty propagation problem which can be solved using different types of computational approaches. Several sampling-based methods, analytical methods and hybrid methods have been developed by researchers in the field of uncertainty quantification and it is necessary to investigate the applicability of these methods to prognostics and health management. Further, several challenges involved in integrating uncertainty quantification techniques into prognostics and health management were outlined. It is clear that further research is necessary to address these challenges and develop a comprehensive framework for uncertainty quantification in prognostics and health management.

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BIOGRAPHIES



Shankar Sankararaman received his B.S. degree in Civil Engineering from the Indian Institute of Technology, Madras in India in 2007 and later, obtained his Ph.D. in Civil Engineering from Vanderbilt University, Nashville, Tennessee, U.S.A. in 2012.

His research focuses on the various aspects of uncertainty quantification, integration, and management in different types of aerospace, mechanical, and civil engineering systems. His research interests include probabilistic methods, risk and reliability analysis, Bayesian networks, system health monitoring, diagnosis and prognosis, decision-making under uncertainty, treatment of epistemic uncertainty, and multidisciplinary analysis. He is a member of the Non-Deterministic Approaches (NDA) technical committee at the American Institute of Aeronautics, the Probabilistic Methods Technical Committee (PMC) at the American Society of Civil Engineers (ASCE), and the Prognostics and Health Management (PHM) Society. Currently, Shankar is a researcher at NASA Ames Research Center, Moffett Field, CA, where he develops algorithms for uncertainty assessment and management in the context of system health monitoring, prognostics, and decision-making.



Kai Goebel is the Deputy Area Lead for Discovery and Systems Health at NASA Ames where he also directs the Prognostics Center of Excellence. After receiving the Ph.D. from the University of California at Berkeley in 1996, Dr. Goebel worked at General Electric's Corporate Research Center in Niskayuna, NY from 1997 to 2006 as a senior research scientist before joining NASA. He has carried out applied research in the areas of artificial intelligence, soft computing, and information fusion and his interest lies in advancing these techniques for real time monitoring, diagnostics, and prognostics. He holds 17 patents and has published more than 250 papers in the area of systems health management.