Improved Railway Track Irregularities Classification by a Model Inversion Approach

René Schenkendorf¹, Beate Dutschk², Katrin Lüddecke³, and Jörn C. Groos⁴

¹,²,³,⁴ Institute of Transportation Systems, German Aerospace Center (DLR e.V.)
Lilienthalplatz 7, 38108 Braunschweig, Germany
rené.schenkendorf@dlr.de
beate.dutschk@dlr.de
katrin.lueddecke@dlr.de
joern.groos@dlr.de

ABSTRACT

Over time railway networks have become complex systems characterized by manifold types of technical components with a broad range of age distribution. De facto, about 50 percent of the life cycle costs of railway infrastructures are made up by direct and indirect maintenance costs. A remedy can be provided by a condition based preventive maintenance strategy leading to an optimized scheduling of maintenance actions taking the actual as well as the expected future infrastructure condition into account. A prerequisite is, however, that the thousands of kilometers of railway tracks are almost continuously monitored. Thus, a promising approach is the usage of low-cost sensors, e.g. accelerometers and gyroscopes, which can be installed on common in-line freight and passenger trains. Due to ambiguous data records a credible classification of railway track irregularities directly from these data is challenging. Alternatively to this pure data-driven approach, in this paper a novel hybrid approach is presented. To this end, a simplified vehicle suspension model is applied for the purpose of railway track condition monitoring by analyzing the dynamic railway track - train interactions. The inversion of the model can be used to recalculate the actual inputs (irregularities) of the monitored system (rail surface) which have caused recorded system responses (dynamic vehicle reactions and acceleration data, respectively). These recalculated inputs are a sound basis of subsequent data-driven condition monitoring analyses. In this preliminary study, a classification algorithm is implemented to identify a simulated railway track irregularity automatically.

1. INTRODUCTION

Roughly speaking, half of the life cycle costs of railway infrastructures are caused by maintenance costs (Gradinariu, 2008). Rail surface irregularities (e.g. Fig.1), in particular, are critical in terms of safety and reliability and call for advanced and cost saving monitoring concepts (Haigermoser, Luber, Rauh, & Gräfe, 2015; Schenkendorf, Groos, & Johannes, 2015). Here, condition based preventive maintenance strategies may help leading to an optimized scheduling of maintenance actions taking into account the actual as well as the prospective infrastructure condition. To implement a seamless condition monitoring of high-frequency, high-capacity railway networks low-cost monitoring systems are mandatory. In detail, micro electro-mechanical systems (MEMS) inertial sensors (accelerometers and gyroscopes) which have the potential to be installed on a large amount of common in-line freight and passenger trains (e.g. (Ward et al., 2011; Molodova, Li, & Dollevoet, 2011; Naganuma, Kobayashi, & Tsunashima, 2014; Weston, Roberts, Yeo, & Stewart, 2015; Quirke, Cantero, OBrien, & Bowe, 2016)) are promising candidates. In this way, the gathered acceleration data may be used on a daily basis to reveal and monitor relevant railway track irregularities by signal processing tools as for instance the Continuous Wavelet Transformation (CWT). Due to ambiguous data records, however, a credible classification of railway track irregularities directly from low-cost sensors acceleration data form in-line trains is challenging. The response of the vehicle to the track irregularities is sensitive to a number of vehicle-specific and partly time varying vehicle parameters (e.g. mass, suspension system, speed, wheel diameter, and position of the measurement device). For instance, a train passing the same critical track segment with different speeds produces different acceleration signals. Due to these unsolved challenges pure data-driven approaches with low-cost sensors on in-line trains are still not commonly implemented in today railway monitoring.

René Schenkendorf et al. This is an open-access article distributed under the terms of the Creative Commons Attribution 3.0 United States License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.
concepts. Alternatively to the pure data-driven approach, in this work a hybrid approach is presented. To this end, a simplified vehicle suspension model, a.k.a. quarter-car model, is applied prior to a data-driven approach for the purpose of railway track condition monitoring. The inverse model can be used to recalculate the actual inputs (irregularities) of the monitored system (rail surface) which have caused recorded system responses (dynamic vehicle reactions and acceleration data, respectively). This approach addresses the unsolved problem how to systematically consider these relevant and variable parameters by the detection of rail track irregularities with low-cost sensors on in-line trains. As shown in this manuscript, these recalculated inputs are a sound basis for track irregularity detection by a following data-driven approach providing, in addition, a valuable input for subsequent studies, e.g. life cost analysis (Rama & Andrews, 2016) and prognostic charts (Saha, Goebel, & Christophersen, 2009; Cocheteux, Voisin, Levrat, & Iung, 2010).

The remainder of this paper is structured as follows. In Section 2 the idea of the novel hybrid approach for rail surface condition monitoring is outlined. Here, in 2.1 the concept of CWT as an essential signal processing tool is shortly summarized. In 2.2 the quarter-car model is derived. The basics of a model inversion strategy to recalculate track irregularities are addressed in Section 2.3. The classification strategy based on recalculated track irregularities is demonstrated in 3. Finally, the conclusion is given in Section 4.

2. HYBRID APPROACH

As previously described, the automatic detection of rail surface failures via low-cost sensor systems is an active research field within the PHM community. In literature, various approaches can be found utilizing acceleration data to assess the rail surface quality by classification and fault detection algorithms (Fig. 2a). These studies, however, are usually based on idealized assumptions, e.g. an unique measurement system installed on a dedicated train with constant speed and masses of the vehicle. Constraints which are rarely met in practice and, in consequence, are one main reason why a credible condition monitoring of the rail surface quality fails to the present day. Alternatively, model-based approaches incorporating expert knowledge of the monitored system can gain helpful insight in terms of fault detection and identification. When implementing model-based concepts in conjunction with data analysis ideas, a hybrid approach is formed (Lee, Ni, Djurdjanovic, Qiu, & Liao, 2006) combining the advantages of both strategies. In this study, a novel hybrid approach is presented. Instead of utilizing a conventional model, an inverse model is applied to recalculate the model inputs, i.e. the rail surface quality (Fig. 2b). In consequence, more credible results can be provided by the classification of these recalculated inputs instead of analyzing the raw acceleration data directly. But before demonstrating the efficiency of the overall concept, the most relevant elements are explained in subsequent.

2.1. Continuous Wavelet Transformation

In any data-driven concept informative features have to be derived from the analyzed data, e.g. geometrical or statistical quantities calculated in the time and/or frequency domain of the signal. In this study, frequency dependent characteristics assigned to the recorded track position are of fundamental importance. Thus, the Continuous Wavelet Transformation is applied to extract these localized frequency features of a signal under investigation. This time-frequency resolution provides essential information of rail surface failures, associated data time spans and localization within the track, respectively. In detail, the mathematical definition of the CWT reads as:

$$W(a, b) := |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t) \Psi^{*} \left( \frac{t-b}{a} \right) dt; \quad a, b \in \mathbb{R}, a \neq 0, \quad (1)$$

where $f(t)$ is the signal under study (e.g. acceleration data, $y$, or recalculated rail surface, $\hat{u}$), $\Psi(\cdot)$ a so-called mother wavelet (here, a mexican hat wavelet is used), $*$ indicates the complex conjugate, $a$ and $b$ are scaling and translation parameters, respectively. For more details, the interested reader is referred to (Teolis, 1998) and references therein.

Depending on $a$ and $b$, wavelet coefficients, $W(a, b)$, can be determined which reveal time-frequency patterns of a signal. These characteristics, sometimes referred as the fingerprint of a signal, can be used as an indicator of the health status of the analyzed system, e.g. the presence or absence of rail surface failures and the condition of other railway assets as well (e.g. (Asada & Roberts, 2013; Molodova, Li, Nunez, & Dollevoet, 2013; Cantero & Basu, 2015)). Technically, in this paper normalized wavelet coefficients (against the signals...
Figure 2. Compared to the conventional data-driven approach (a) the proposed hybrid approach (b) combines data-driven concepts with model-based ideas.

(2) total energy Eq.\((2)\) are used to ensure a better comparability.

\[ E = \int_{-\infty}^{\infty} |f(t)|^2 dt. \]  

Typically, CWT is directly applied to the raw data for the purpose of condition monitoring. In this study, however, CWT is combined with a model-based approach as outlined in the next sections.

2.2. Mechanistic Model

Beside the data-driven concepts, first-principle / mechanistic models are excellent tools to take account for expert knowledge of the monitored system and to gain valuable insight for a detailed diagnosis. For instance, the response of the vehicle dynamic can be simulated by a mechanical vehicle suspension system. In the simplest case a single axis movement is modeled (Fig. 3) known as quarter-car models (Imine, 2011; Naganuma et al., 2014). The governing equation set of this quarter-car model is presented in its state-space form:

\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du \]  

where \( u \in \mathbb{R}^{n_u} \) and \( y \in \mathbb{R}^{n_y} \) are the system inputs and the outputs, respectively. The system states are given by \( x \in \mathbb{R}^{n_x} \). The system matrices are known as the dynamic matrix \( A \), the input matrix \( B \), the output matrix \( C \), and the feedthrough matrix \( D \).

In case of the quarter-car model the corresponding matrices, assuming \( x = [\ddot{x}_s, \ddot{x}_{us}, \dot{x}_s, \dot{x}_{us}]^\top \) and \( y = \ddot{x}_s \), are:

\[
A = \begin{bmatrix}
-c/m_s & -c/m_s & -k_s/m_s & -k_s/m_us \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(4)

\[
B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(5)

\[
C = \begin{bmatrix}
-c/m_s & -c/m_s & -k_s/m_s & -k_s/m_us \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(6)

\[
D = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(7)

Here, the system includes the sprung mass, \( m_s \), and the unsprung mass of the vehicle, \( m_{us} \), which are connected by a linear spring and damper with the stiffness coefficient, \( k_s \).
and the damping constant, \( c \), respectively. The rail surface is considered as the system input, \( u(t) \), and is transmitted by a spring \((k_{us})\) to the unsprung mass (Fig.3). The numerical values of the model parameters are given in Table 1.

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_s )</td>
<td>9875 [kg]</td>
</tr>
<tr>
<td>( m_{us} )</td>
<td>1100 [kg]</td>
</tr>
<tr>
<td>( k_s )</td>
<td>( 2.13 \times 10^6 ) [N/m]</td>
</tr>
<tr>
<td>( k_{us} )</td>
<td>( 1.42 \times 10^5 ) [N/m]</td>
</tr>
<tr>
<td>( c )</td>
<td>( 1.20 \times 10^4 ) [N/s/m]</td>
</tr>
</tbody>
</table>

2.3. Model Inversion

The special feature of this study is to not directly apply the derived vehicle model for condition monitoring but its inverse. In general, the basic concept of model inversion aims at using the recorded output data of the system under study and to reconstruct the underlying inputs by systems theory principles. Various model inversion strategies exist, see (Czop, Mendrok, & Uhl, 2011; Schenkendorf & Groos, 2015) and references therein. In this work, the inverse simulation approach is applied extending the original model by a feedback control loop (Buchholz & v. Grünhagen, 2007; Murray-Smith, 2011). For the purpose of illustration the state-space model (Eq.(3)) is transferred into the transfer function notation first, i.e. applying Laplace Transformation:

\[
G(s) = C(sI - A)^{-1}B + D
\]  

In general, the transfer function, \( G(s) \), represents the input/output behavior of the system:

\[
G(s) = \frac{Y(s)}{U(s)}
\]

A straightforward inversion of the transfer function is in most practical cases not feasible as the resulting inverse transfer function becomes non-causal. Alternatively, the corresponding closed-loop system (Fig.4) results in:

\[
G(s) = \frac{U(s)}{Y(s)} = \frac{K^c \cdot U(s)}{U(s) + K^c \cdot Y(s)}
\]

\[
\lim_{K^c \to \infty} G^1(s) = \frac{U(s)}{Y(s)}
\]

(11)

(Here, \( K^c = 1000 \).) Transferred back into the time-domain, the reconstructed input reads as:

\[
u(t) \approx \hat{u}(t) = \int_0^t g^1(\tau - t)y(\tau)d\tau
\]

(12)

As demonstrated in the subsequent section, these recalculated inputs \( \hat{u} \) (rail surfaces) are a sound basis for condition based maintenance, i.e. the machine-aided detection of rail surface failures triggering optimized maintenance actions.

3. Demonstration

After essential aspects of the proposed hybrid concept have been presented, their gainful interaction is illustrated via a preliminary simulation study and compared with the traditional approach, i.e. pure data-driven analysis of acceleration data. First, informative features of the signal under study have to be extracted. In this work, CWT is applied and the resulting wavelet coefficients (Eq.(1)) are rearranged as feature vectors. Here, each feature vector represents a track segment of 5 m length. These feature vectors train a classification algorithm. In subsequent, a Support Vector Machine (SVM) (Bishop, 2008) is used which, in theory, classifies new incoming data according to the underlying track quality. (A radial basis kernel is used.) Considering the standard approach, i.e. evaluating the acceleration data directly, ambiguous data records due to variation in vehicle parameters (e.g. mass, speed) make a proper classification difficult. Assuming a rail segment of 240 m length with corrugation from meter 60 to 160, four different scenarios are modeled and analyzed, i.e. acceleration data of the unsprung mass at low speed (40 km/h) and higher speed (160 km/h) compared to acceleration data of the sprung mass at low speed (40 km/h) and higher speed (160 km/h). Simulating a corrugated rail segment, i.e. a periodic pattern on the rail surface, the resulting CWT analysis is shown in Figs.(5a-5d). Obviously, the resulting wavelet coefficients are sensitive to the position of the installed measurement system (e.g. unsprung or sprung mass) and the traveling speed of the train (e.g. low or high). A proper classification based on these findings is challenging.

Alternatively, when using acceleration data to reconstruct the rail surface first the CWT analysis reveals similar patterns compared to the original rail surface failures. In detail, the wavelet coefficients of the original (Fig.5e) and of the reconstructed rail surface (Fig.5f) are compared. Obviously, there are no significant differences detectable. It should be stressed, that independent of the sensor position (unsprung
Figure 5. CWT analysis: Scalograms representing $|W(\alpha, b)|$ (Eq. (1)) with low, medium, and high numeric values. When analyzing acceleration data, $f(t) := y(t) = \ddot{x}_{us}(t)$ or $f(t) := y(t) = \ddot{x}_s(t)$, the wavelet coefficients vary in their values at the different scenarios shown in (a)-(d): 1) Higher speed correlates to higher frequency contribution (c)-(d) and lower scale, respectively. 2) Acceleration data of the sprung mass show in addition a relevant eigenmode contribution of the system, see the high scale range of (b) & (d). However, evaluating the rail surface geometry alternatively results in similar wavelet coefficients for the simulated ($f(t) := u(t)$) and the recalculated rail surface ($f(t) := \ddot{u}(t)$) as shown in (e)-(f).
or sprung mass) or the speed (low or high) the reconstructed rail surface and the wavelet coefficients are equivalent.

In consequence, using a feature vector based on wavelet coefficients of the reconstructed and non-ambiguous inputs the actual classification step becomes easier. For the purpose of demonstration, a track segment of 500 m length is simulated showing two different kinds of rail irregularities: (1) a corrugated part from meter 50 to 150; (2) long-periodic irregularities from meter 250 to 450. A properly trained SVM is used to identify these irregularities based on the recalculated input (rail surface), i.e. incorporating simulated sprung mass acceleration data indirectly. As illustrated in Fig.6, except for the transition zones the critical parts of the rail segment are correctly identified.

4. Conclusion

In this study it is shown how a model inversion strategy can be usefully combined with machine learning techniques forming a hybrid approach. In this way, the original gathered acceleration data are first transferred back into an unambiguous rail surface profile which is used for further analysis and classification purposes. Preliminary results of this hybrid approach are derived by a simplified simulation study. Here, under ideal assumptions (i.e. perfect measurement data and no process noise) the critical track segments are identified correctly. In future, the proposed concept will be extended in the following way: (1) incorporating real (non-simulated) data; (2) extending the quarter-car model to a full car-model to distinguish between rail surface irregularities and structure-borne noise; (3) applying special forms of Kalman Filtering for the purpose of model inversion and rail surface reconstruction, respectively.

Acknowledgment

The financial support by German Aerospace Center (DLR e.V., Cologne) under grant TrackScan is gratefully acknowledged.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>system dynamic matrix</td>
</tr>
<tr>
<td>B</td>
<td>system input matrix</td>
</tr>
<tr>
<td>C</td>
<td>system output matrix</td>
</tr>
<tr>
<td>D</td>
<td>system feedthrough matrix</td>
</tr>
<tr>
<td>E</td>
<td>energy of a signal</td>
</tr>
<tr>
<td>G</td>
<td>transfer function</td>
</tr>
<tr>
<td>( G^\dagger )</td>
<td>inverse transfer function</td>
</tr>
<tr>
<td>W</td>
<td>wavelet coefficient matrix</td>
</tr>
<tr>
<td>a</td>
<td>wavelet scaling parameter</td>
</tr>
<tr>
<td>b</td>
<td>wavelet translation parameter</td>
</tr>
<tr>
<td>c</td>
<td>damping constant</td>
</tr>
<tr>
<td>f</td>
<td>signal/data vector</td>
</tr>
<tr>
<td>( k_s )</td>
<td>spring stiffness coefficient sprung mass</td>
</tr>
<tr>
<td>( k_{us} )</td>
<td>spring stiffness coefficient unsprung mass</td>
</tr>
<tr>
<td>( m_s )</td>
<td>sprung mass of the vehicle</td>
</tr>
<tr>
<td>( m_{us} )</td>
<td>unsprung mass of the vehicle</td>
</tr>
<tr>
<td>u</td>
<td>system input</td>
</tr>
<tr>
<td>( \hat{u} )</td>
<td>recalculated system input</td>
</tr>
<tr>
<td>x</td>
<td>system states</td>
</tr>
<tr>
<td>( x_s )</td>
<td>position of the sprung mass</td>
</tr>
<tr>
<td>( \dot{x}_s )</td>
<td>velocity of the sprung mass</td>
</tr>
<tr>
<td>( \ddot{x}_s )</td>
<td>acceleration of the sprung mass</td>
</tr>
<tr>
<td>( x_{us} )</td>
<td>position of the unsprung mass</td>
</tr>
<tr>
<td>( \dot{x}_{us} )</td>
<td>velocity of the unsprung mass</td>
</tr>
<tr>
<td>( \ddot{x}_{us} )</td>
<td>acceleration of the unsprung mass</td>
</tr>
<tr>
<td>y</td>
<td>system output</td>
</tr>
<tr>
<td>( \Psi )</td>
<td>mother wavelet function</td>
</tr>
<tr>
<td>( \Psi^* )</td>
<td>complex conjugate wavelet function</td>
</tr>
</tbody>
</table>

References


GRIN Publishing GmbH.
Schenkendorf, R., & Groos, J. C. (2015). Global sensitiv-


**Biographies**

**René Schenkendorf** René received his Dipl.-Ing. and Dr.-Ing. in Engineering Cybernetics from the Otto-von-Guericke-University Magdeburg, Germany in 2007 and 2014, respectively. From 2007 until 2012 he had been a Ph.D. student at the Max Planck Institute for Dynamics of Complex Technical Systems in Magdeburg, Germany. From 2013 until 2016 he was with the German Aerospace Center at the Institute of Transportation Systems in Braunschweig, Germany. Since 2016 René is the head of the Process Systems Engineering group at the Institute of Energy and Process Systems Engineering at the Technische Universität Braunschweig, Germany. His current research interests include data analysis, statistical process control, uncertainty analysis, and modeling in PHM.

**Beate Dutschk** Beate received her M.Sc. in Systems Engineering and Engineering Cybernetics from the Otto-von-Guericke University Magdeburg, Germany, in 2015. Since 2015 she is with the German Aerospace Center at the Institute of Transportation Systems in Braunschweig, Germany. Her current research interests include data analysis for condition monitoring of railway assets.
Katrin Lüddecke  Katrin studied electrical engineering and received both her diploma from the Technische Universität Braunschweig, Germany, and her M.Sc. from the Royal Institute of Technology (KTH) in Stockholm, Sweden, in 2006. Since 2006 she is with the German Aerospace Center at the Institute of Transportation Systems in Braunschweig, Germany. There, she leads the working group Asset and System Monitoring. Her research interests include data acquisition and data analysis for vehicle-borne positioning and condition monitoring of railway assets.

Jörn C. Groos  Jörn received his diploma in geophysics from the University of Karlsruhe, Germany, in 2007 and his Dr. rer. nat. from the faculty of physics of the Karlsruhe Institute of Technology in 2010. Since 2014 he is with the German Aerospace Center at the Institute of Transportation Systems in Braunschweig, Germany. His current research interests include data acquisition, data analysis, and pattern recognition for condition monitoring of railway tracks.