

Failure Prognostics with Missing Data Using Extended Kalman Filter

Wlamir Olivares Loesch Vianna¹, and Takashi Yoneyama²

¹ *EMBRAER S.A., São José dos Campos, São Paulo, 12227-901, Brazil*
wlamir.vianna@embraer.com.br

² *ITA - Instituto Tecnológico de Aeronáutica, São José dos Campos, São Paulo, 12228-900, Brazil*
takashi@ita.br

ABSTRACT

Failure prognostics can provide benefits in operation and maintenance of equipments by predicting when the component is going to fail and consequently acting at the most appropriate time. In several situations degradation estimations are sparse or missing estimations are present at collected data. Considering these situations, a failure prognostics method was proposed considering the usage of the extended version of the Kalman filter. This method was analyzed with hydraulic system reservoir levels indication collected from four different aircrafts. In this study a prognostic model was estimated by the filter and then future values of hydraulic level as well as the remaining useful life interval were obtained considering a set of Monte Carlo simulations and a failure probability distribution approximation. Results evidenced the benefit of this method to properly prognose the system.

1. INTRODUCTION

The Kalman filter was initially proposed in (Kalman, 1960) and is a recursive filter using noisy and even incomplete measurements to estimate the states of a linear system in the time domain. Inputs of the algorithm are the system information and optionally some knowledge from the controls on the plant if it is known. The existence of two independent noises is considered, one perturbs the information from system (measurement noise) and the other is mixed with linear operator (process noise).

Several applications of the Kalman filter and its derivations in prognostics and health monitoring (PHM) relates to fault prognosis and remaining useful life (RUL) estimation. An example is included in (Gomes, Leao, Vianna, Galvão, & Yoneyama, 2012) where a linear Kalman filter is used with

historical degradation estimations to estimate the rate probability distribution of the degradation increase. Next, Monte Carlo (MC) simulations are performed to build the RUL probability distributions. Another application example is included in (Leao, 2011) where an alternative of the MC is proposed using the Unscented transform. Benefits from the usage of these types of techniques compared to other traditional methods such as linear or polynomial regression include a better consideration of transitory wear dynamics (Lim & Mba, 2015) and spurious data. Also, the implementation of these techniques requires some parameters definitions such as the estimation noise which influences the performance of results. Recommendations of these parameters can be found in (Leao, 2011) and (Vianna, Souza Ribeiro, & Yoneyama, 2015).

Alternative solutions found in literature include the usage of Particle filter techniques for non linear prognostic problems. Examples include (Daigle & Goebel, 2013) and (Orchard & Vachtsevanos, 2009). Its main advantage is to make possible the approximation of the entire probability distribution, but as it requires Monte Carlo methods, an increased number of simulations are required and consequently more computational capacity is required.

For several applications, such as airlines, degradation estimation intervals are not fixed and missing data may be present at the collected data for prognostics implementation. In such situations, the usage of the Kalman filter, as in (Gomes et al., 2012), becomes more difficult since its recursive approach considers only fixed intervals. This work proposes a method to solve this problem by means of the extended version of the Kalman filter (EKF) introduced in (Jazwinski, 1970). The EKF is probably the most popular Bayesian estimation for nonlinear systems. It is based on the linearization of the model equations to allow the application of the LKF to nonlinear systems. The main purpose of using this method is to demonstrate how the least computational expensive non linear bayesian estimation method can solve the RUL estimation

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problem of not fixed time intervals robustly and efficiently considering an illustrative example of aircraft hydraulic reservoir levels.

The remaining sections are organized as follows: section 2 describes the proposed methodology; section 3 presents a case study considering some aircraft hydraulic level operational data and section 4 contains the conclusion.

2. FAILURE PROGNOSTICS METHODOLOGY

The implementation of the Kalman filter for failure prognostics considering fixed time intervals and no missing data can be found in (Gomes et al., 2012). In this application, a linear prognostic model is considered and is described in Eq. (1) and Eq. (2).

$$x_{wear_k} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x_{wear_{k-1}} + w_{wear_{k-1}} \quad (1)$$

$$z_{wear_k} = [1 \quad 0] x_{wear_k} + v_{wear_k} \quad (2)$$

In which x_{wear_k} represents the dynamic of the degradation at interval k . $x_{wear_k}(1)$ is the actual value of the degradation and $x_{wear_k}(2)$ is its rate of decrease (or increase) per time interval. Observes that this model represents a linear trend and if one wish to build a higher order model, one or more states should be included. The parameter $w_{wear_{k-1}}$ represents the process vector noise and v_{wear_k} the measurement vector noise. The parameter z_{wear_k} represents the current degradation estimation.

For the implementation of the LKF at each operation step, first it is necessary to define the covariance matrices of process noise Q_{wear_k} and measurement noise R_{wear_k} . Eq. (3) and Eq. (4) show a proposal for their definitions.

$$Q_{wear_k} = \begin{bmatrix} 0 & 0 \\ 0 & q_{wear} \end{bmatrix} \quad (3)$$

$$R_{wear_k} = [r_{wear}] \quad (4)$$

The parameter q_{wear} must be defined during the filter implementation. Higher its value, higher will be the estimation variance of degradation rate and faster will be the response for degradation rate changes identification. An example of how this parameter influences estimations will be illustrated in section 3. Similarly, the parameter r_{wear} must also be defined during the filter implementation and is related to the degradation estimation variance. Examples of how this parameter definition influences the results are also included in section 3.

Once the prognostic model is found, it can be used for future estimations of the degradation and consequently the remaining useful life (RUL) estimation. In this work a set of Monte Carlo (MC) simulations are executed considering the prog-

nostic model estimation parameters as well as its covariance matrix also found during the filter implementation. Time to failure samples are then found considering a certain degradation threshold and finally, RUL distribution fit into a certain probability distribution model (i.e. Weibull).

Considering the situations where degradation estimations intervals are not fixed, or missing data are present at collected data, the current method can no longer be applied. To solve this problem a nonlinear model is proposed with the inclusion of the time interval between last degradation collected and the current one as a model input. Eq. (5) and Eq. (6) describe this model.

$$x_{wear_k} = f_k(x_{wear_{k-1}}, \Delta t_k, w_{wear_{k-1}}) = x_{wear_{k-1}}(1) + x_{wear_{k-1}}(2)\Delta t_k + w_{wear_{k-1}} \quad (5)$$

$$z_{wear_k} = g_k(x_{wear_k}, v_{wear_k}) = x_{wear_k}(1) + v_{wear_k} \quad (6)$$

In which Δt_k is the degradation estimation interval here represented as model input. Notice that this model is not linear and the LKF can no longer be used. To solve that, the extended Kalman filter is invoked. The model linearization is based on the partial derivatives matrix (Jacobian) described in Eq. (7).

$$\left. \frac{\partial f(x_{k-1}, u_{k-1})}{\partial x_{k-1}} \right|_{x_{k-1}=\hat{x}_{k-1}^-} = \begin{bmatrix} 1 & \Delta t_k \\ 0 & 1 \end{bmatrix} \quad (7)$$

In which \hat{x}_{k-1}^- is the *a priori* (predicted) state estimate at Instant $k-1$. The resulting linearized model is described in Eq. (8) and Eq. (9).

$$x_{wear_k} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{wear_{k-1}} + \begin{bmatrix} \hat{x}_{wear_{k-1}}^-(2) \\ 0 \end{bmatrix} \Delta t_k \quad (8)$$

$$z_{wear_k} = [1 \quad 0] x_{wear_k} \quad (9)$$

An important observation relates to the process noise $w_{wear_{k-1}}$ definition, which in this case are not fixed and depends on the input variable Δt_k . Considering that, the process noise matrix Q_{wear_k} becomes

$$Q_{wear_k} = \begin{bmatrix} 0 & 0 \\ 0 & \Delta t_k q_{wear} \end{bmatrix} \quad (10)$$

in which q_{wear} is the same as presented in Eq. (3).

Now that the linearized prognostic model is defined considering time intervals not fixed, the EKF can be implemented and the model parameters estimated. Next section contains a case study application considering the prognostics of an hydraulic system reservoir level.

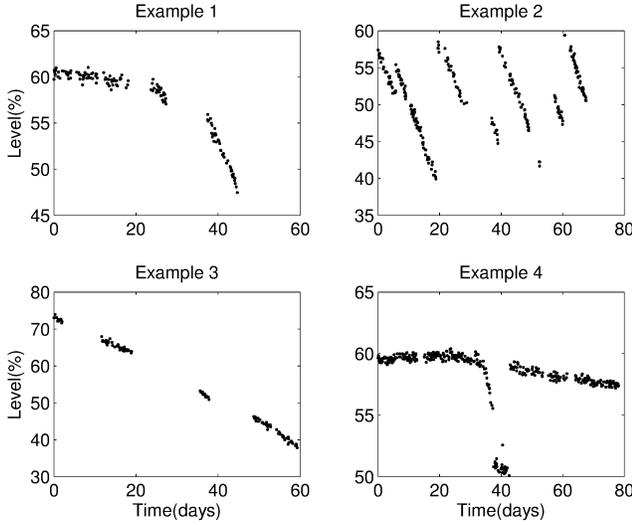


Figure 1: Aircraft Hydraulic System Levels Examples.

3. AIRCRAFT HYDRAULIC SYSTEM LEVEL MONITORING CASE STUDY

In this case study a degradation estimation method described in (Vianna & Malere, 2014) is considered. In this method, a normalized hydraulic reservoir level from an aircraft hydraulic system is estimated by means of a physics model considering the fluid properties, system dynamic (actuators, accumulators, etc) and reservoir/system fluid capacities. The difference between the actual estimated level and the low level limit is defined as the actual degradation and its variance represents the system leakage. Fig. 1 shows the levels estimations collected from operational data of four different aircrafts used in this case study.

In example 1, it is observed two intervals with missing data and a increase in leakage in time. Example 2 shows an aircraft with intense leakage and several hydraulic fluid filling tasks (sudden increase in level) without fixing the source of leakage. Example 3 shows a constant leakage with also some missing data over certain time intervals and example 4 shows a sudden excessive leakage increase with its identification and repair after few days and a fluid filling task (again with a sudden increase in level).

The states estimation of the prognostic model considering the EKF implementation is exhibited next. First in Fig. 2, the estimation hydraulic levels ($x_{wear_k}(1)$ in Eq. [1]) are shown together with the observation levels. In this example a value of $1e^{-1}$ is considered for q_{wear} and $1e^{-4}$ for r_{wear} . At the intervals with missing data, the prognostic model is not updated and degradation rate is fixed until next observation. Also, to properly address the fluid filling tasks, the filter covariance matrix was reseted and states reinitialized after an abrupt level

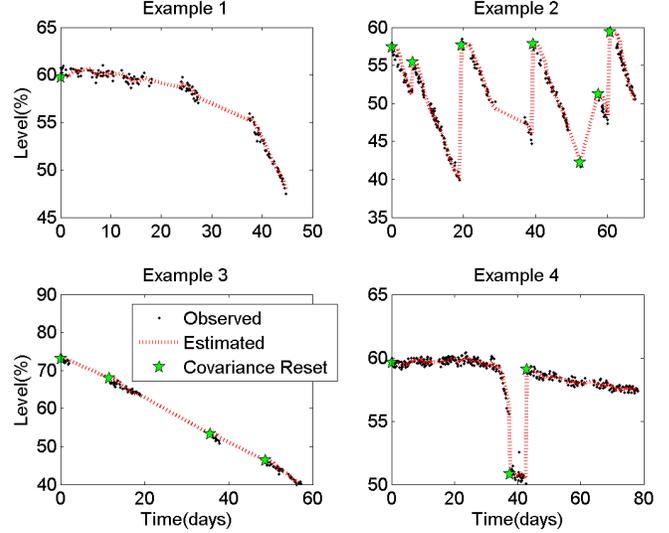


Figure 2: Hydraulic Levels Estimation.

variance. The occurrence of this resetting procedure is also shown in Fig. 2.

Notice from this result that the algorithm could identify the variations in degradations since estimations are close to observations even for those situations of abrupt changes. Also, the filling processes and other variances were also properly identified and covariances and states reseted at the right situations. Finally, it is possible to observe from the results that estimations presented lower variances compared to observed degradations (as expected). Notice that this variance can present variations as different values of the parameter r_{wear} is assigned at the filter implementation.

Results of the degradation rate estimations ($x_{wear_k}(2)$ in Eq. [1]), here represented as the hydraulic system leakage, are presented in Fig. 3. Also in this case, covariance and states resets are exhibited.

In this implementation, at each reset, the degradation rate ($x_{wear_k}(2)$) is assigned a null value (no leakage) and if the system present any degradation rate, the filter incrementally observes degradations and identifies its value. This process is better identified in example 3 from Fig. 3 where the hydraulic system presents a constant degradation rate (leakage) and due to some missing data, covariance and states are reseted and a null value to degradation rate is assigned. As new observations and filtering steps are implemented, increased leakage estimations are observed. The variance of the degradation rate estimation as well as how fast the filter can take to estimate sudden changes in its value depends on the parameter q_{wear} value assigned. Fig. 4 shows the results obtained for several values of this parameter considering example 4.

From these results, it is possible to see that higher values of

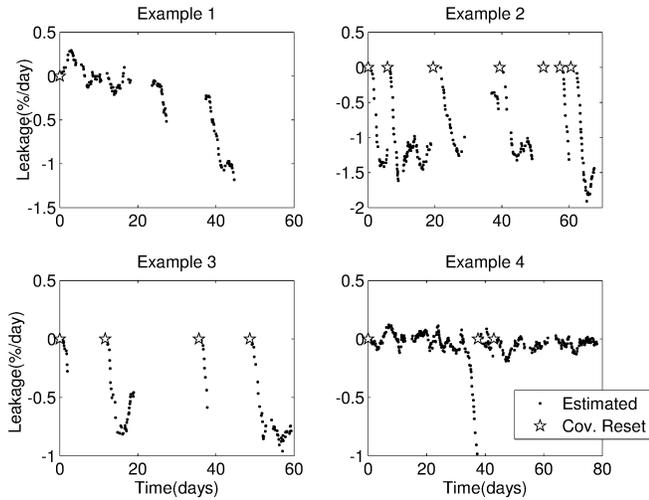


Figure 3: Leakage Estimations.

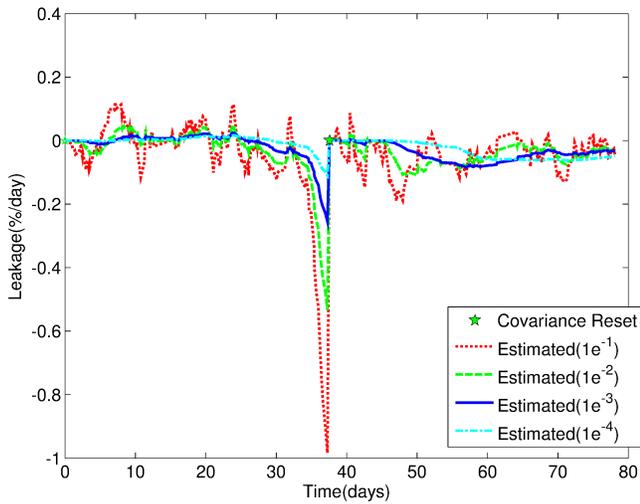


Figure 4: Process Noise Influence in Leakage Estimations for Example 4.

q_{wear} result in higher variance of the degradation rate, but on the hand sudden variations in degradations are identified faster as seen between days thirty and forty for example 4. Notice that the higher values of q_{wear} could not even identify the increased degradation rate before repair and fluid filling around day forty.

A similar analysis was made considering variations in the r_{wear} parameter considering also example 4. Fig. 5 shows this result.

It is possible to observe that increased values of r_{wear} resulted in lower variances in degradation estimations ($x_{wear_k}(1)$).

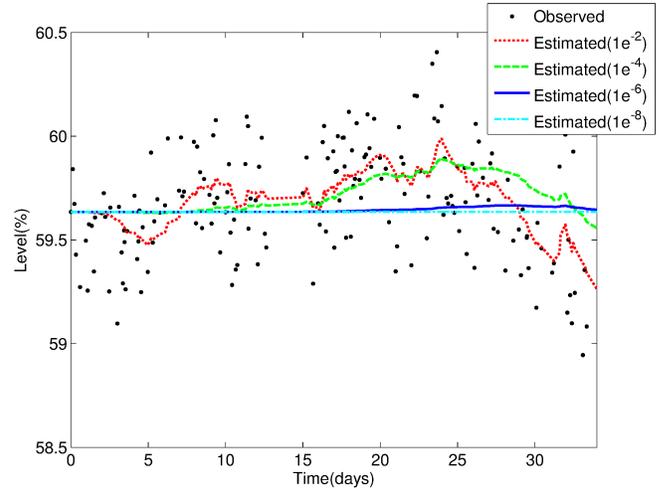


Figure 5: Measurement Noise Influence in Level Estimations for Example 4 and first 30 days of analysis.

Once the prognostics model is obtained, future estimations in degradation can be obtained and consequently RUL estimations at each observation interval. Fig. 6 shows the observed and estimated RUL considering a low level limit of 40%. In this case a Weibull probability distribution fitted by means of a maximum likelihood estimation (MLE) algorithm was considered. The Weibull distribution plays an important role in the analysis of reliability and survival data and its great flexibility makes it suitable in numerous applications (Blischke & Murthy, 2000). Successful applications for RUL distributions fitting in PHM analysis include (Alves, Oliveira Bizarria, & Galvao, 2009) and (Vianna et al., 2015). Also 20.000 MC simulations and a confidence interval of 85% were considered. In this figure, the y axis represents the RUL in days and the x axis the estimation date, whose origin is at the actual failure, here described as the time that the hydraulic level reached its limit. The blue solid line represent the true RUL and red dotted line the estimated RUL. Also the confidence interval is plotted (red solid bars) considering the Weibull distribution. Notice that the confidence interval is not symmetric as the time to failure histogram from the MC simulations and consequently the Weibull distributions are also not symmetric. For more details around this interval estimation method, see (Alves et al., 2009).

It is possible to observe from this result that although RUL estimation were higher than actual values in Fig. 6, all confidence intervals results contained the actual RUL estimation, so the prognostic model could predict correctly the actual failure considering the given confidence.

4. CONCLUSION

This study proposed a prognostic method considering degradation estimations intervals not fixed and missing data using

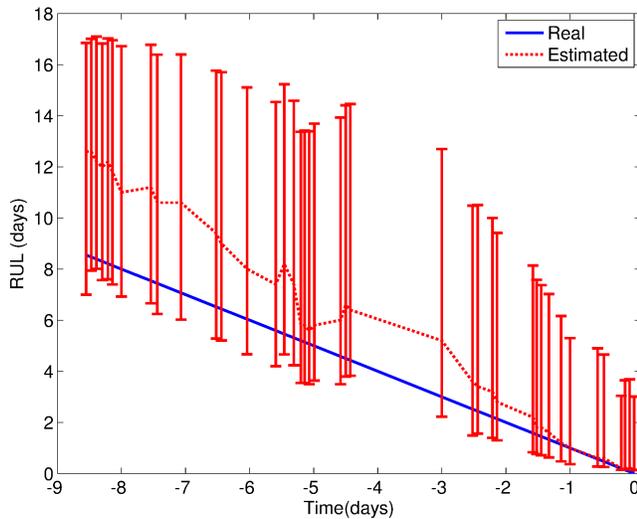


Figure 6: Hydraulic Low Level Prognostics for Example 3.

the extended version of the Kalman filter. In this approach, a discrete state space model for the degradation trend was built in which the time interval between each degradation sample is considered as input. Considering that this model is not linear, the EKF had to be invoked which required the current model linearization. A case study was conducted with field data considering an aircraft hydraulic system level monitoring algorithm. Results were presented for four different examples collected from operational data of four different aircrafts. Analysis of degradation and leakage estimation was considered as well as the influence of these parameters considering different set up parameters such as the process noise and measurement noise. Also, RUL intervals were estimated considering an hydraulic low level event. This estimation was based on a Weibull distribution obtained from Monte Carlo simulations of the prognostic model found during the filtering step. It was possible to conclude that given the data set, the proposed method could properly prognose the system failure.

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BIOGRAPHIES



Wlamir Olivares Loesch Vianna holds a bachelor's degree on Mechanical Engineering (2005) from Universidade de São Paulo (USP), Brazil, and Master Degree on Aeronautical Engineering (2007) from Instituto Tecnológico de Aeronáutica (ITA), Brazil. He is with Empresa Brasileira de Aeronáutica S.A (EMBRAER), São José dos Campos, SP, Brazil, since 2007. He works as a Development Engineer of a R&T group at EMBRAER focused on PHM technology applications in aeronautical systems.



Takashi Yoneyama is a Professor of Control Theory with the Electronic Engineering Department of ITA. He received the bachelor's degree in electronic engineering from Instituto Tecnológico de Aeronáutica (ITA), Brazil, the M.D. degree in medicine from Universidade de Taubaté, Brazil, and the Ph.D. degree in electrical engineering from the University of London, U.K. (1983). He has more than 300 published papers, has written four books, and has supervised more than 70 theses. His research is concerned mainly with stochastic optimal control theory. He served as the President of the Brazilian Automatics Society in the period 2004–2006.