Bayesian Approach for the Lethargy Coefficient Estimation in the Probabilistic Creep-Fatigue Life Model

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ABSTRACT

The researches of Prognostics and Health Management (PHM) have been important in the field of engineering. The crack is propagated by high temperature and stress in power plants, vehicle engines and etc. The defect and damage are also accumulated. Therefore, it is necessary for design of creep-fatigue life about various structures and etc. In this study, probabilistic life design based on Zhurkov life model was performed using the lethargy coefficient under the variety of temperatures and stress conditions. For this work, the integration life equation was derived using Zhurkov life model. The deterministic lethargy coefficient is calculated to using the reference of the Small Punch (SP)-Creep test and tensile-shear test data about steel material (rupture stress and rupture time). Markov Chain Monte Carlo (MCMC) sampling method based on Bayesian framework is employed for estimating the lethargy coefficient and considering its uncertainties. As a result, predicted creep-fatigue life was observed that it was considerably decreased in accordance with increasing temperature and stress conditions relatively. This life model is reasonable through comparing with conventional creep-fatigue life data.

Key Words: Prognostics and Health Management (PHM), Creep-fatigue life, Lethargy coefficient, Zhurkov model, Markov Chain Monte Carlo (MCMC), Bayesian framework

1. INTRODUCTION

Nowadays, Prognostics and Health Management (PHM) have been applied to the field of engineering. The crack propagation by high temperature and stress is occurred due to accumulating defect and damage continuously in power plants, vehicle engines and so on. Therefore, it needs the design of creep-fatigue life about various structures under operating conditions. For this work, the estimation of parameters is required using finite data set in structural analysis or the health management of structures. In the early stage of structural design, material properties are obtained from various experiments. It is affect to the validity of structural analysis. Meta-model model which can replace the finite element analysis (FEA) is employed to saving computational cost but it is that there are also inherent uncertainties due to experiment error and the lack of data. For quantifying the uncertainties of material parameters or model coefficients, stochastic and statistical manners are employed. In the health management, degradation parameters of physical model in the deteriorating structures are estimated using monitoring data over times for the prognostics of creep-fatigue life.

Bayesian framework for fatigue model determination, updating and averaging using trans-dimensional Markov Chain Monte Carlo (MCMC) simulation (C. Andrieu et al., 2003) is presented. Uncertainties are introduced by model choice, mechanism modeling, model parameter, and response measures are systematically included. Additional response measures are used to update model probabilities and parameter distributions associated with each of the models simultaneously via one trans-dimensional MCMC simulation in the general state space. The results of Bayes factors serve as a reference for model comparisons and determinations (X. Guan et al., 2010). This framework is also outlined for the parameter estimation that arises during the uncertainty quantification in the numerical simulation as well as in the prognosis of the structural performance. The parameters are estimated in the form of posterior distribution conditional on the provided data. During the numerical implementation, MCMC method is employed, which is a modern computational technique for the efficient and straightforward estimation of parameters (Choi et al., 2010). X. Guan et al. compared with two probabilistic prognosis updating schemes. One is based on the classical Bayesian approach and the other is based on newly

developed maximum relative entropy (MRE) approach. The algorithm performance of the two models is evaluated using a set of recently developed prognostics-based metrics. Various uncertainties from measurements, modeling, and parameter estimations are integrated into the prognosis framework as random input variables for fatigue damage of materials. Measures of response variables are then used to update the statistical distributions of random variables and the prognosis results are updated using posterior distributions. MCMC technique is employed to provide the posterior samples for model updating in the framework. Experimental data are used to demonstrate the operation of the proposed probabilistic prognosis methodology (X. Guan et al., 2011). The methodology to quantify the uncertainty in fatigue crack growth prognosis applied to structures with complicated geometry and subjected to variable amplitude multi-axial loading. The Gaussian process surrogate model is used to replace the expensive finite element analysis. The various sources of uncertainty include, three different types of modeling errors are included in analysis such as crack growth model error, discretization error and surrogate model error. The different types of uncertainty are incorporated into the crack growth prediction methodology to predict the probability distribution of crack size (S. Sankararaman et al., 2011). In other research, the parameters of the proposed creep-fatigue model were estimated using a standard Bayesian regression approach. It has been performed Bayesian analysis using the MCMC sampling method. The results have shown a reasonable fit between the experimental data and the proposed probabilistic creep-fatigue life assessment models (F. Ibisoglu et al., 2015).
In this study, probabilistic creep-fatigue life which based on Zhurkov (S. N. Zhurkov, 1965) model is suggested using stochastically and statistically estimated lethargy coefficient. the creep-fatigue life model was derived using Zhurkov life model and this model was deterministically validated with the reference of creep-fatigue life data (Park et al., 2011). For this process, firstly, lethargy coefficient which is relative to the failure of materials has to be obtained with rupture time and stress from quasi static tensile-shear test and Small Punch (SP) creep test. These experiments are performed using HS40R and X20CrMoV121 steel respectively (Sin et al., 2011; Park et al., 2011). However, lethargy coefficient has uncertainties due to experiment errors and the variation of material properties by inherent defects. Bayesian approach was employed for estimating its coefficient of the creep-fatigue life model using Markov Chain Monte Carlo (MCMC) sampling method. Once the samples are obtained, one can proceed to the posterior predictive inference on the creep-fatigue life.

2. EXPERIMENTS

2.1 Quasi Static Tensile-Shear Test for HS40R

In this study, HS40R (high strength steel sheet) is widely used for the body frame of automobiles and so on. For calculating the lethargy coefficient about this material, rupture time and stress are obtained from quasi static tensile-shear test. Using specimens in Figure 1 were carried out the tensile test with the controlling displacement method with INSTRON 8516. The tensile velocity was set to 2 mm/min and displacements were measured with a contacted strain gauge. Spot welding condition of the specimen is depending on KS B 0850. Chemical compositions and obtained material properties of the HS40R are represented respectively in the Table 1 and Table 2.

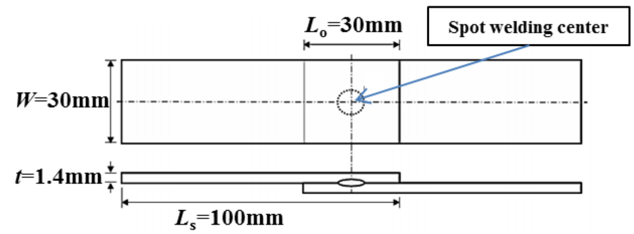


Figure 1. Quasi static tensile-shear test specimen

Table 1. Chemical composition of HS40R (w_t %)

C	Si	Mn	P
0.0876	0.0065	0.7407	0.1241
S	Ni	Al	Fe
0.0036	0.0091	0.3577	Bal.

Table 2. Mechanical properties of HS40R

Material	σ_t (MPa)	σ_y (MPa)	Elongation (%)
HS40R	416.5	286.1	39

2.2 Fatigue Test of HS40R Steel

From the quasi static tensile-shear test, rupture stress and time were obtained for calculating lethargy coefficients. In order to compare with predicted life, fatigue test is preceded with dynamic fatigue tester as INSTRON 8516. The frequency of alternative load was set to 10Hz and the cyclic load is controlled under stress ratio condition (R ≒ 0). The behaviors of fatigue crack propagation under the spot welding with 6kA and 2 lap are observed by using direct current potential drop method (DCPDM). It use the displacement current by behavior of fatigue crack propagation. The fatigue life of the HS40R is represented in the Table 3

Table 3. The summary of experiments of HS40R

HS40R Steel (300K)	Welding current (kA)	Rupture stress (MPa)	Rupture time (sec)	Alternate load (MPa)	Experimental fatigue life (Cycle)
2 lap	6	229.5	60.6	113.5	52750.0
				112.5	162440
				112.0	291160
				111.7	408190

2.3 SP-Creep Test of X20CrMoV121 Steel

X20CrMoV121 steel is widely used for boiler pipe in thermal power generation, or steam valve such as high temperature and pressure state. Chromium (Cr) steel (9~12%) is recommended for thermal power generations boiler header. In this study, small-punch (SP) creep test was conducted using the steel with Cr content 10%. Chemical compositions and obtained material properties of the X20CrMoV121 are represented respectively in the Table 4 and Table 5.

Table 4. Chemical compositions of X20CrMoV121 (w_t%)

C	Si	Mn	P	S
0.19	0.19	0.48	0.011	0.003
Ni	Cr	Mo	V	Fe
0.66	10.40	0.86	0.26	Bal.

Table 5. Mechanical properties of X20CrMoV121

Material	σ_t (MPa)	σ_y (MPa)	Elongation (%)
X20CrMoV121	798.5	572.8	19.4

When performing the SP-creep test, creep load is subjected to ceramic (Si_3N) rigid ball. The central displacement of specimens was measured according to time. The spec of the creep tester is that temperature error is in ± 1℃, high temperature range is from 550℃ to 650℃ and load range is in the 20~100kg. During the test, central displacement of materials is measured using the linear variable differential transformer (LVDT). It can be precisely measured until 1μm.

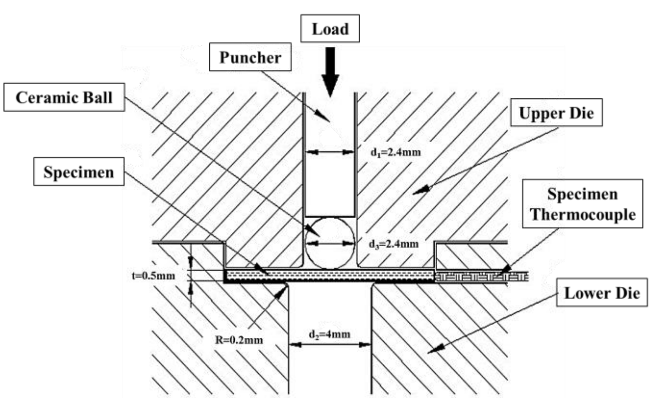


Figure 3. Expansion of test portion

SP-creep test was conducted under the high temperature conditions as 898K and creep load conditions as 108.3~173.3MPa. However, rupture stress and rupture time can't be found directly in creep test for calculating the lethargy coefficient. Generally, rupture time and stress's relation can be expressed $t_r = A\sigma^{-m\sigma t}$ (A is constant) in the tensile creep test. We found the lethargy coefficient when it occurs similar result to creep life using these relationships. The SP-creep life is represented according to temperature and stress conditions in Table 6.

Table 6. Results of creep test for X20CrMoV121

Material	Temperature (K)	Actual stress (MPa)	Experimental rupture time (hours)
X20CrMoV121 Steel	898	108.3	369.43
		130.0	128.00
		151.6	54.066
		173.3	18.914

3. CREEP-FATIGUE LIFE MODEL

Zhurkov drew the empirical fatigue life model under uniform stress and temperature conditions. This model is assumed that the failure occurs when the grid of atoms is removed from stable state. The probability which an atom is removed by thermal vibration for steady time from the position of grid is considered under the mentioned hypothesis. The static fatigue equation of Zhurkov is transformed into the relation of dynamic fatigue. The creep-fatigue life model is represented as Equation (1).

$$\int_0^L \frac{dt}{t_0 e^{\frac{U_0 - \gamma\sigma(t)}{kT(t)}}} = 1 \qquad (1)$$

Here, $\sigma(t)$ and $T(t)$ are stress and absolute temperature are changed with arbitrary time function respectively. U_0 is internal energy; t_0 is time constant as 10^{-13} sec and k is Boltzman constant. γ is lethargy coefficient that the material characteristics are involved in accordance with defects and metallography.

When creep and fatigue are respectively occurred, life can be obtained using general life model but it's difficult to predict the life about occurring creep and fatigue concurrently. Generally, fatigue is generated by repeated load under uniform temperature. On the other hands, creep is occurred by the temperature effect under uniform stress. Therefore, when creep and fatigue is concurrently generated, we can consider the relation of stress ($\sigma = \bar{\sigma} + \hat{\sigma}\cos\omega_1 t$) and temperature ($T = \bar{T} + \hat{T}\cos\omega_2 t$) through transforming into time functions. Zhurkov's creep-fatigue life model can be changed with this equation (2). Here, $\bar{\sigma}$ and $\bar{T}$ are mean stress and mean temperature respectively. $\hat{\sigma}$ and $\hat{T}$ are the amplitude of alternate stress and alternate temperature, these parameters are substituted in Equation (1). It is represented as Equation (2).

$$\int_0^L \frac{dt}{t_0 e^{\frac{U_0 - \gamma\bar{\sigma}(1+(\hat{\sigma}/\bar{\sigma})\cos\omega_1 t)}{k\bar{T}(1+(\hat{T}/\bar{T})\cos\omega_2 t)}}} = 1 \qquad (2)$$

However, equation (2) can't be integrated directly, if ($\hat{T}/\bar{T}$ << 1) equation changed as follows equation (3)

$$\frac{1}{t_0 e^{\frac{U_0 - \gamma\bar{\sigma}}{k\bar{T}}}} \int_0^L e^{\frac{\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{\sigma}}{\bar{\sigma}}\cos\omega_1 t} e^{\frac{\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{T}}{\bar{T}}\cos\omega_2 t} dt = 1 \qquad (3)$$

Equation (3) can be changed as Equation (6) using Cauchy-Schwarz inequality conditions as Equation (4) and the Bessel functions as Equation (5),

$$\int_a^b f(x)g(x)dx \le \left[\int_a^b f^2(x)dx \int_a^b g^2(x)dx\right]^{\frac{1}{2}} \quad (4)$$

$$\int_{L_1}^{L_2} e^{x\cos x}dx = (L_2 - L_1)I_0(x) \quad (5)$$

$$I_0(0) = 1$$
$$I_0(x) = e^x / \sqrt{2\pi x}$$
$$I_0(2x) = e^{2x} / \sqrt{2\pi 2x} = I_0^2(x)\sqrt{\pi x}$$

$$\frac{1}{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}} L\left[I_0\left(2\frac{\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{\sigma}}{\bar{\sigma}}\right)I_0\left(2\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{T}}{\bar{T}}\right)\right]^{\frac{1}{2}} \ge 1 \quad (6)$$

Integrated life (L) equation can be derived as follows Equation (7).

$$L \ge \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{\left[I_0\left(2\frac{\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{\sigma}}{\bar{\sigma}}\right)I_0\left(2\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{T}}{\bar{T}}\right)\right]^{\frac{1}{2}}} \quad (7)$$

Fatigue and creep are respectively affected by stress and temperature. If fatigue will be occurred by alternate load under uniform temperature, $\hat{T}$ is 0 and $I_0(0) = 1$. Therefore, fatigue life cycle (L_f) is following as Equation (8) (Yang et al., 1997).

$$L_f \ge \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{I_0\left(2\frac{\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{\sigma}}{\bar{\sigma}}\right)^{\frac{1}{2}}} \quad (8)$$

Equation (8) is represented as follows Equation (9)

$$L_f \ge \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{I_0(2x)^{\frac{1}{2}}} = \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{(I_0^2(x)\sqrt{\pi x})^{\frac{1}{2}}} \quad \text{where, } x = \frac{\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{\sigma}}{\bar{\sigma}}$$

$$= \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{I_0(x)} = \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{I_0(\frac{\gamma\bar{\sigma}}{k\bar{T}})} \quad (9)$$

Finally, the fatigue life model is derived as follows Equation (10).

$$L_f \ge t_0\sqrt{2\pi}\sqrt{\frac{\gamma\hat{\sigma}}{k\bar{T}}}\frac{e^{\left(\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}\right)}}{e^{\left(\frac{\gamma\hat{\sigma}}{k\bar{T}}\right)}}$$
$$L_f = t_0\sqrt{2\pi}\sqrt{\frac{\gamma\hat{\sigma}}{kT}}e^{\left(\frac{U_0-\gamma(\bar{\sigma}+\hat{\sigma})}{kT}\right)} \quad (10)$$

We have to consider the frequency of alternate stress. fatigue life cycle can be obtained by the multiplication of fatigue life (L_f) and frequency (f), $N=L_f \bullet f$. We can predict the fatigue life cycle using Equation (11) (Yang et al., 1997; Park et al., 2011).

$$N_f = ft_0\sqrt{2\pi}\sqrt{\frac{\gamma\hat{\sigma}}{kT}}e^{\left(\frac{U_0-\gamma(\bar{\sigma}+\hat{\sigma})}{kT}\right)}(cycle) \quad (11)$$

In case of creep life, alternate load is 0. Its life (L_c) can be derived from Equation (7) as follows Equation (12) (Yang et al., 1997).

$$L_c \ge \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{I_0\left(2\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{T}}{\bar{T}}\right)^{\frac{1}{2}}} \quad (12)$$

Equation (12) is represented as follows Equation (13)

$$= \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{(I_0^2(x)\cdot\sqrt{\pi x})^{\frac{1}{2}}} \quad \text{where, } x = \frac{(U_0-\gamma\bar{\sigma})}{k\bar{T}}\frac{\hat{T}}{\bar{T}}$$
$$= \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{I_0(x)} = \frac{t_0 e^{\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}}}{I_0(\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{T}}{\bar{T}})}$$

$$L_c = t_0 \exp\frac{U_0-\gamma\bar{\sigma}}{kT} / I_0\left(\frac{U_0-\gamma\bar{\sigma}}{k\bar{T}}\frac{\hat{T}}{\bar{T}}\right) \quad (13)$$

The creep life model [Equation (14)] is also derived from Equation (13) using Bessel functions.

$$L_c = t_0\sqrt{2\pi}\sqrt{\frac{(U_0-\gamma\bar{\sigma})\hat{T}}{k\bar{T}^2}}e^{\left(\frac{(U_0-\gamma\bar{\sigma})(\bar{T}-\hat{T})}{k\bar{T}^2}\right)}(\text{sec}) \quad (14)$$

3.1. Creep-fatigue life using lethargy coefficient

The lethargy coefficient is represented with defect constant of material characteristic; therefore, many tensile tests are needed to determine γ. In this study, determining γ, results from simple quasi static tensile-shear tests performed by Park et al. (1998) were used. We computed the lethargy coefficient and creep-fatigue life using the rupture stress and rupture time under uniform temperature and loading conditions. The lethargy coefficient can be obtained by using Equation (15) (Song et al., 2004).

$$\gamma = \frac{U_0}{\sigma_u}(1-\eta) \quad (15)$$

Lethargy coefficient is proportional to the internal energy over the rupture stress and (1-η). η is variable for calculating lethargy coefficient that is expressed as equation (16).

Where,
$$\eta = \frac{\ln\left(\frac{t_r}{t_0}\right)}{\frac{U_0}{kT}}\left(1-\frac{\ln\left(\frac{U_0}{kT}-\ln\left(\frac{t_r}{t_0}\right)\right)}{\ln\left(\frac{t_r}{t_0}\right)\left[1-\left(\frac{U_0}{kT}-\ln\left(\frac{t_r}{t_0}\right)\right)^{-1}\right]}\right) \quad (16)$$

3.2 Deterministic creep-fatigue life

Deterministic creep-fatigue life were obtained using Zhurkov life model about HS40R and X20CrMoV121 materials. Lethargy coefficient was calculated using constant and rupture parameters in Table 7 and Table 8. After that, fatigue life was predicted using Equation (11) in accordance with alternate stress conditions. We also estimated the creep rupture time using Equation (14) according to alternating temperature conditions. One can see that predicted life was approximately corresponded with experimental data. Predicted fatigue life cycle and creep rupture time are represented in the Table 8 and Table 9.

Table 7. Constants for lethargy coefficient

Material	T	300K
	t_0	10^{-13} sec
	k	0.008384 kJ/mole•k
HS40R	U_0	418.4 kJ/mole
X20CrMoV121	U_0	344.4 kJ/mole

Table 8. Results of fatigue life for HS40R (2 lap / 6kA)

Rupture stress (MPa)	Rupture time (sec)	Lethargy coefficient (kJ/mole • mm^2/N)	Alternate stress (MPa)	Predicted Fatigue life (Cycle)	Experimental Fatigue life (Cycle)
229.5	60.6	1.4501	113.5	51929.0	52750.0
			112.5	163770	162440
			112.0	290840	291160
			111.7	410490	408190

Table 9. Results of creep test for X20CrMoV121

Temperature (K)	Lethargy coefficient (kJ/mole • mm^2/N)	Actual stress (MPa)	Predicted rupture time (Hours)	Experimental rupture time (Hours)
898	0.1325	108.3	422.33	369.43
	0.1824	130.0	119.62	128.00
	0.2087	151.6	41.103	54.066
	0.2223	173.3	16.278	18.914

4. BAYESIAN ESTIMATION OF LETHARGY COEFFICIENTS USING MARKOV CHAIN MONTE CARLO (MCMC)

For estimation of the lethargy coefficient of Zhurkov creep-fatigue model, Bayes' rule is used as follows Equation (17) (Bayes, 1763).

$$p(\boldsymbol{\theta} \mid \mathbf{y}) \propto L(\mathbf{y} \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta}) \quad (17)$$

Here, $L(\mathbf{y}|\boldsymbol{\theta})$ is the likelihood of observed data **y** conditional on the given parameters $\boldsymbol{\theta}$, $p(\boldsymbol{\theta})$ is the prior distribution of $\boldsymbol{\theta}$, and $p(\boldsymbol{\theta}|\mathbf{y})$ is the posterior distribution of $\boldsymbol{\theta}$ conditional on **y**. The equation states that our degree of belief on the parameter $\boldsymbol{\theta}$ is expressed as posterior *pdf* in right of the given data **y**. As more data are provided, the posterior distribution is again used as a prior at the next step, and the values are update to more confident information. This is called Bayesian updating. The Procedure to obtain posterior distribution $p(\boldsymbol{\theta}|\mathbf{y})$ consists of proper definition of probability distribution for the likelihood and prior respectively (Leem et al., 2011). For estimating posterior distribution of the lethargy coefficient, Markov model is widely used in various fields in which sequence of the data is very meaningful. Markov chain consists of Markov model defines probability of posterior event given the prior events. The idea of MCMC basically the same as the Markov model in that it defines posterior position of the sampling point based on the prior information of the sampled points. Most important technique can be employed in MCMC, the Metropolis-Hastings (M-H) algorithm which is the most simplified MCMC method can be performed using Equation (18) by the following steps.

1. Initialise $x^{(0)}$

2. For i = 0 to nm-1

- Sample $u \sim U_{[0,1]}$

- Sample $x^* \sim q(x^* | x^{(i)})$

$$\text{if } u < A\left(x^{(i)}, x^*\right) = \min\left\{1, \frac{p(x^*)q(x^{(i)} \mid x^*)}{p(x^{(i)})q(x^* \mid x^{(i)})}\right\} \quad (18)$$

$$x^{(i+1)} = x^*$$

else

$$x^{(i+1)} = x^{(i)}$$

In these steps, x_0 is the initial value of an unknown parameter to estimate, nm is the number of iterations or samples, U is a uniform distribution, $p(x)$ is the posterior distribution (target PDF), and $q(x^*|x_i)$ is an arbitrary chosen proposal distribution which is used when a new sample x^* is to be drawn conditional on the current point x_i. Uniform or normal distribution at the current point is the most common choices for the proposal distribution. Success and failure of the algorithm significantly depends on a proper design of the proposal distribution. In order to illustrate this, a target distribution of x is considered (C. Andrieu et al, 2003). In this study, Bayesian method was employed for estimating the lethargy coefficient. Posterior distribution was also estimated through MCMC simulation assuming proposal distribution as normal distribution. The formulation of Bayes' rule is represented as our engineering problem in Equation (19) and Equation (20). Likelihood function can be represented with Equation (21). It's the observed life data of creep and fatigue life from Equation (11) and Equation (14) respectively.

$$p(\gamma \mid N) \propto L(N \mid \gamma)\, p(\gamma) \tag{19}$$

$$p(\gamma \mid N) \propto \left(\frac{1}{\sqrt{2\pi}\sigma_{std}} \right)^k \exp\left[-\frac{1}{2\sigma_{std}^2} \sum_{i=1}^{k} \left(N_{estimation}^i - N_{actual}^i \right)^2 \right] \times p(\gamma) \tag{20}$$

$$L(N \mid \gamma) = \left(\frac{1}{\sqrt{2\pi}\sigma_{std}} \right)^k \exp\left[-\frac{1}{2\sigma_{std}^2} \sum_{i=1}^{k} \left(N_{estimation}^i - N_{actual}^i \right)^2 \right] \tag{21}$$

With only 10,000 iterations, the sampling result follows the target distribution quite well. Estimated posterior distributions of lethargy coefficients were corresponded with actual lethargy coefficients as follows Figure 4 and Figure 5.

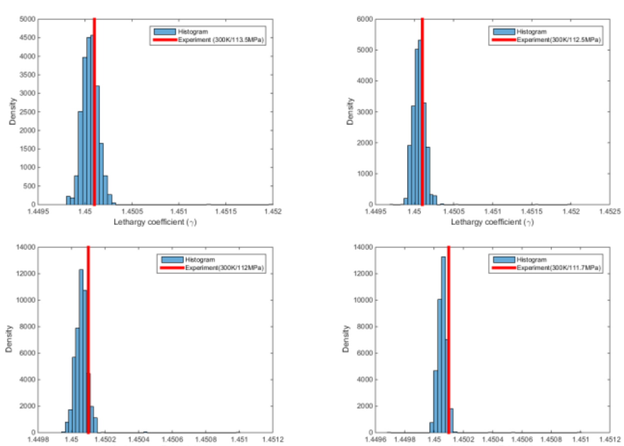


Figure 4. Posterior distribution of γ (Fatigue life)

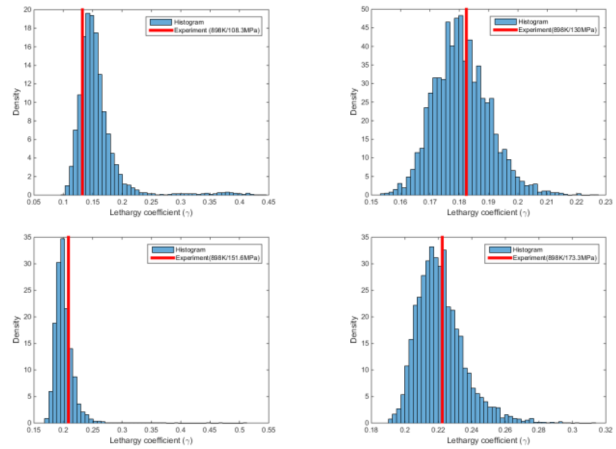


Figure 5. Posterior distribution of γ (Creep life at 898K)

5. RESULTS OF PROBABILISTIC CREEP-FATIGUE LIFE

The probability density function (*pdf*) distributions of creep-fatigue life were predicted using estimated the lethargy coefficient by MCMC simulation. In case of fatigue life, the *pdf* distributions were predicted according to alternate stress at room temperature (300K). Results are corresponded with actual life cycle. In addition, the *pdf* of creep life was estimated in accordance with alternate stress at 898K. The results also agreed with actual life time. The *pdf* of estimated fatigue life is shown as narrow deviations because lethargy coefficient is identical according to alternate stress. However, in creep model is different to lethargy coefficients according to alternate stress at the high temperature. Thus, the *pdf* of creep life is relatively shown as high deviations.

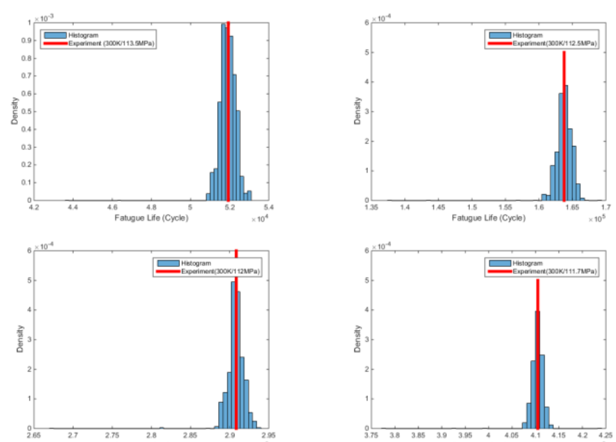
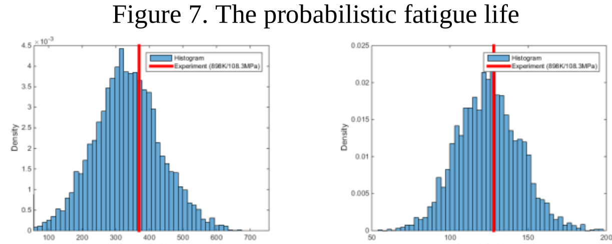


Figure 7. The probabilistic fatigue life



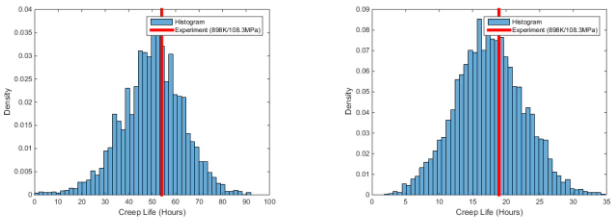


Figure 8. The probabilistic creep life at 898K

6. CONCLUSIONS

Zhurkov life model based on probabilistic creep-fatigue life models is derived under the cyclic stress and temperature. Lethargy coefficients were calculated using experimental data of rupture stress and time. Using this parameter, the life model was deterministically validated with actual life data. For considering the inherent uncertainty of lethargy coefficient of life models, the degree of belief on the model parameters is expressed through a posterior probability distribution in light of the observed data combined with the prior knowledge. Bayesian inference manner was employed and its coefficient was also estimated using MCMC which is used to obtain the posterior predictive distribution. One can see that estimated posterior distributions are corresponded with experimental data. As a result of probabilistic creep-fatigue life using estimated the lethargy coefficient is satisfied with actual value. From this result, Bayesian approach is proved to be useful means for the uncertainty quantification of the unknown parameters in the practical engineering problem. This manner is also useful in the field of prognostics.

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NOMENCLATURE

γ	Lethargy coefficient (kJ/mole • mm^2/N)
k	Boltzman constant (kJ/mole • K)
σ	Stress (MPa)
σ_{std}	Standard deviation
σ_t	Tensile stress (MPa)
σ_y	Yield stress (MPa)
σ_r	Rupture stress (MPa)
$\bar{\sigma}$	Mean stress (MPa)
$\hat{\sigma}$	Alternating stress (MPa)
f	Frequency (Hz)
I_0	Bessel function
L	Life time (sec)
N	Life cycle (cycle)
t	Time (sec)
t_0	Life coefficient (sec)
t_r	Rupture time (sec)
T	Absolute temperature (K)
U_0	Internal energy (kJ/mole)
$\boldsymbol{\theta}$	Probabilistic parameter
$L(\mathbf{y}\|\boldsymbol{\theta})$	Likelihood function
$p(\boldsymbol{\theta}\|\mathbf{y})$	Posterior distribution
$p(\boldsymbol{\theta})$	Prior distribution
$q(\mathbf{x})$	Marginal probability density function

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